Linear wave shaping:

The process where by the form of a non-sinusoidal signal is altered by transmission through a linear network is called linear wave shaping.

LOW-PASS *RC CIRCUIT :*

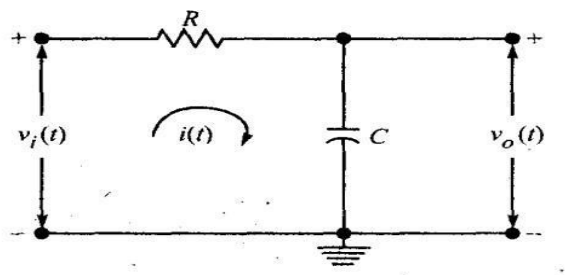


Figure 1.1 The low-pass *RC* circuit.

A low-pass circuit transmits only low-frequency signals and attenuates or stops high-frequency signals

At zero frequency, the reactance of the capacitor is infinity (i.e. the capacitor acts as an open circuit) so the entire input appears at the output

So the output is the same as the input, i.e. the gain is unity

As the frequency increases the capacitive reactance decreases and so the output decreases. At very high frequencies the capacitor virtually acts as a short-circuit and the output falls to zero.

Sinusoidal Input:

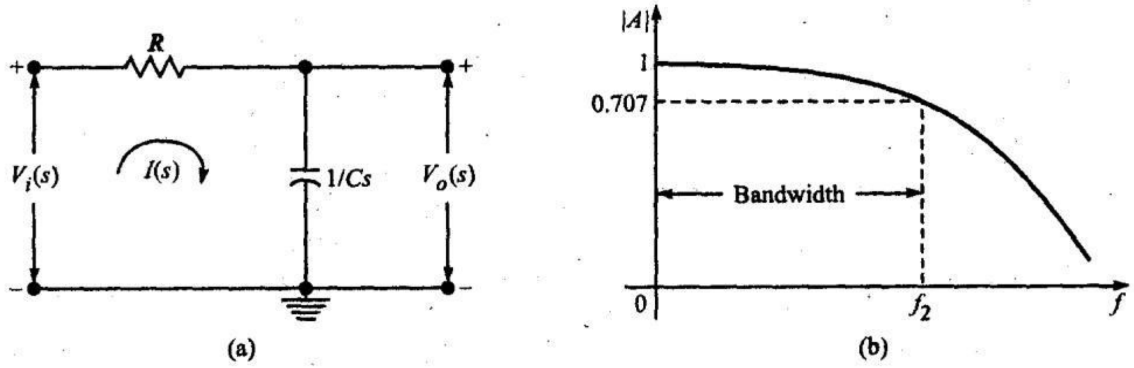


Figure 1.2 (a) Laplace transformed low-pass *RC* circuit and (b) its frequency response.

The gain versus frequency curve of a low-pass circuit excited by a sinusoidal input is shown in Figure 1.2(b).
This curve is obtained by keeping the amplitude of the input sinusoidal signal constant and varying its frequency and noting the output at each frequency.

At low frequencies the output is equal to the input and hence the gain is unity. As the frequency increases, the output decreases and hence the gain decreases.

The frequency at which the gain is l/√2 (= 0.707) of its maximum value is called the cut-off frequency

For a low-pass circuit, there is no lower cut-off frequency. It is zero itself

The upper cut-off frequency is the frequency (in the high-frequency range) at which the gain is 1/√2 . i-e- 70.7%, of its maximum value. The bandwidth of the low-pass circuit is equal to the upper cut-off frequency f2 itself.

For the network shown in Figure 1.2(a), the magnitude of the steady-state gain A is

$$A = \frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{Cs}}{R + \frac{1}{Cs}} = \frac{1}{1 + RCs} = \frac{1}{1 + j\omega RC} = \frac{1}{1 + j2\pi fRC}$$

$$\therefore \quad |A| = \frac{1}{\sqrt{1 + (2\pi fRC)^2}}$$

At the upper cut-off frequency f_2, $|A| = \frac{1}{\sqrt{2}}$

$$\therefore \quad \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1 + (2\pi f_2 RC)^2}}$$

Squaring both sides and equating the denominators,

$$2 = 1 + (2\pi f_2 RC)^2$$

$\therefore$ The upper cut-off frequency, $f_2 = \frac{1}{2\pi RC}$.

So

$$A = \frac{1}{1 + j\frac{f}{f_2}} \quad \text{and} \quad |A| = \frac{1}{\sqrt{1 + \left(\frac{f}{f_2}\right)^2}}$$

The angle θ by which the output leads the input is given by

$$\theta = \tan^{-1}\frac{f}{f_2}$$

Step-Voltage Input:

A step signal is one which maintains the value zero for all times $t < 0$, *and maintains the value* V *for all times* $t > 0$. *The transition between the two voltage levels takes place at* $t = 0$

Thus, in Figure 1.3(a), $v_i = 0$ immediately before $t = 0$ *(to be referred to as time* $t = 0-$*) and* $vi = V$, *immediately after* $t = 0$ *(to be referred to as time* $t = 0+$*).*

if the capacitor is initially uncharged, when a step input is applied, since the voltage across the capacitor cannot change instantaneously, the output will be zero at $t = 0$, *and then, as the capacitor charges, the output voltage rises exponentially towards the steady-state value* V *with a time constant RC*

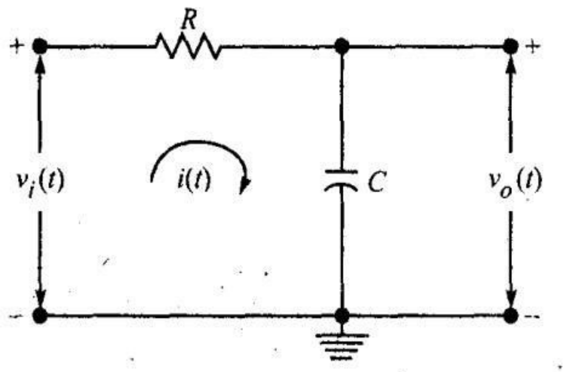


Figure 1.1 The low-pass *RC* circuit.

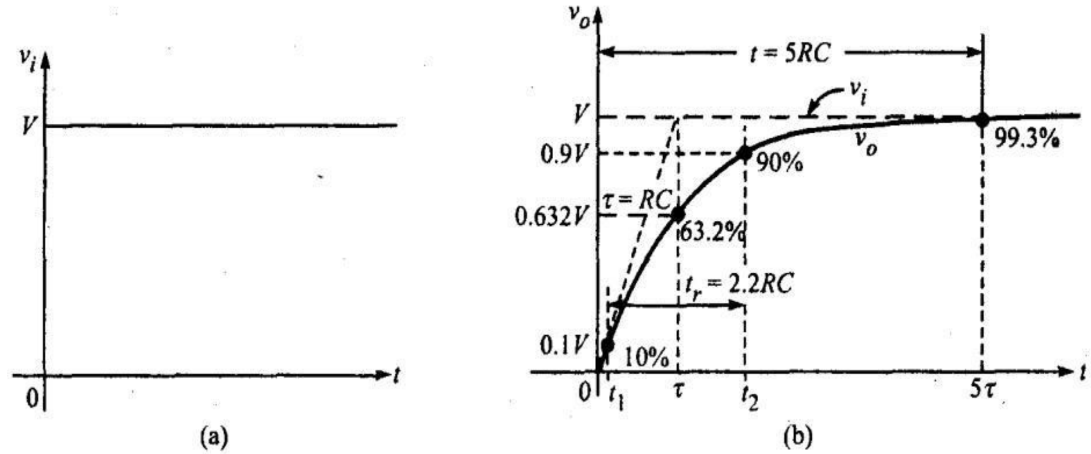


Figure 1.3 (a) Step input and (b) step response of the low-pass *RC* circuit.

Let V’ be the initial voltage across the capacitor. Writing KVL around the loop

$$v_i(t) = Ri(t) + \frac{1}{C}\int i(t)\,dt$$

Differentiating this equation,

$$\frac{dv_i(t)}{dt} = R\frac{di(t)}{dt} + \frac{1}{C}i(t)$$

Since $v_i(t) = V$, $\frac{dv_i(t)}{dt} = 0$

$\therefore$ $$0 = R\frac{di(t)}{dt} + \frac{1}{C}i(t)$$

Taking the Laplace transform on both sides,

$$0 = R\,[sI(s) - I(0^+)] + \frac{1}{C}I(s)$$

$\therefore$ $$I(0^+) = I(s)\left(s + \frac{1}{RC}\right)$$

The initial current $I(0^+)$ is given by

$$I(0^+) = \frac{V - V'}{R}$$

$\therefore$ $$I(s) = \frac{I(0^+)}{s + \frac{1}{RC}} = \frac{V - V'}{R\left(s + \frac{1}{RC}\right)}$$

and $$V_o(s) = V_i(s) - I(s)R = \frac{V}{s} - \frac{(V - V')R}{R\left(s + \frac{1}{RC}\right)} = \frac{V}{s} - \frac{V - V'}{s + \frac{1}{RC}}$$

Taking the inverse Laplace transform on both sides,

$$v_o(t) = V - (V - V')e^{-t/RC}$$

where V' is the initial voltage across the capacitor ($V_{initial}$) and V is the final voltage (V_{final}) to which the capacitor can charge.

So, the expression for the voltage across the capacitor of an RC circuit excited by a step input is given by

$$v_o(t) = V_{final} - (V_{final} - V_{initial})e^{-t/RC}$$

If the capacitor is initially uncharged, then $v_o(t) = V(1 - e^{-t/RC})$

Expression for rise time

When a step signal is applied, the rise time t_r *is defined as the time taken by the output voltage waveform to rise from 10% to 90% of its final value:*

It gives an indication of how fast the circuit can respond to a discontinuity in voltage.

Assuming that the capacitor in Figure 1.1 is initially uncharged, the output voltage shown in Figure 1.3(b) at any instant of time is given by

$$v_o(t) = V(1 - e^{-t/RC})$$

At $t = t_1$, $v_o(t) \neq 10\%$ of $V = 0.1\,\text{V}$

$$\therefore \quad 0.1V = V(1 - e^{-t_1/RC})$$

$$\therefore \quad e^{-t_1/RC} = 0.9 \quad \text{or} \quad e^{t_1/RC} = \frac{1}{0.9} = 1.11$$

$$\therefore \quad t_1 = RC \ln (1.11) = 0.1RC$$

At $t = t_2$, $v_o(t) = 90\%$ of $V = 0.9\,\text{V}$

$$\therefore \quad 0.9V = V(1 - e^{-t_2/RC})$$

$$\therefore \quad e^{-t_2/RC} = 0.1 \quad \text{or} \quad e^{t_2/RC} = \frac{1}{0.1} = 10$$

$$\therefore \quad t_2 = RC \ln 10 = 2.3RC$$

$$\therefore \quad \text{Rise time, } t_r = t_2 - t_1 = 2.2RC$$

This indicates that the rise time t_r *is proportional to the time constant RC of the circuit. The larger the time constant, the slower the capacitor charges, and the smaller the time constant, the faster the capacitor charges.*

Relation between rise time and upper 3-dB frequency

We know that the upper 3-dB frequency (same as bandwidth) of a low-pass circuit is

$$f_2 = \frac{1}{2\pi RC} \quad \text{or} \quad RC = \frac{1}{2\pi f_2}$$

$$\therefore \quad \text{Rise time, } t_r = 2.2RC = \frac{2.2}{2\pi f_2} = \frac{0.35}{f_2} = \frac{0.35}{\text{BW}}$$

Thus, the rise time is inversely proportional to the upper 3-dB frequency.

The *time constant (τ= RC) of a circuit is defined as the time taken by the output to rise to 63.2% of the amplitude of the input step. It is same as the time taken by the output to rise to 100% of the amplitude of the input step, if the initial slope of rise is maintained.*

Pulse Input

The pulse shown in Figure 1.4(a) is equivalent to a positive step followed by a delayed negative step as shown in Figure 1 .4(b).

So, the response of the low-pass *RC circuit to a pulse* for times less than the pulse width *tp is the same as that for a step input and is given by* $v_0(t) = V(l - e^{-t/RC})$.

The responses of the low-pass RC circuit for time constant RC » tp, RC smaller than t_p *and RC very small compared to* t_p *are shown in Figures 1.5(a), 1.5(b), and 1.5(c) respectively.*

If the time constant *RC of the circuit is very large, at the end of the pulse, the output voltage will be* $V_p(t) = V(1 - e^{-tp/RC})$*, and the output will decrease to zero from this value with a* time constant *RC as shown in Figure 1.5(a).*

Observe that the pulse waveform is distorted when it is passed through a linear network. The output will always extend beyond the pulse width tp, because whatever charge has accumulated across the capacitor C during the pulse cannot leak off instantaneously.

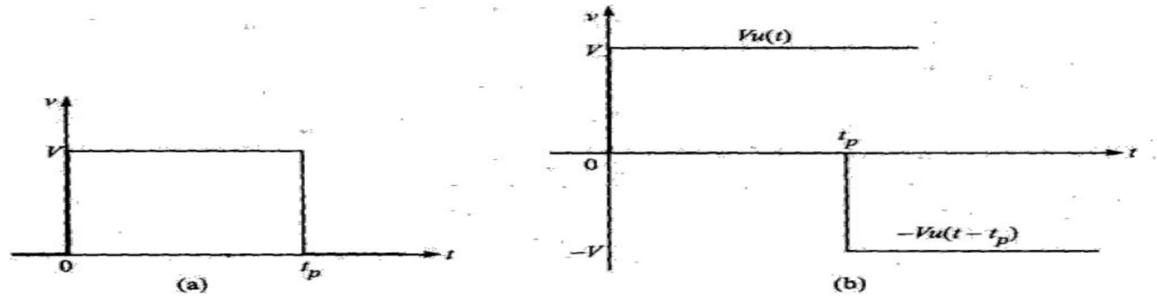


Figure 1.4 (a) A pulse and (b) a pulse in terms of steps.

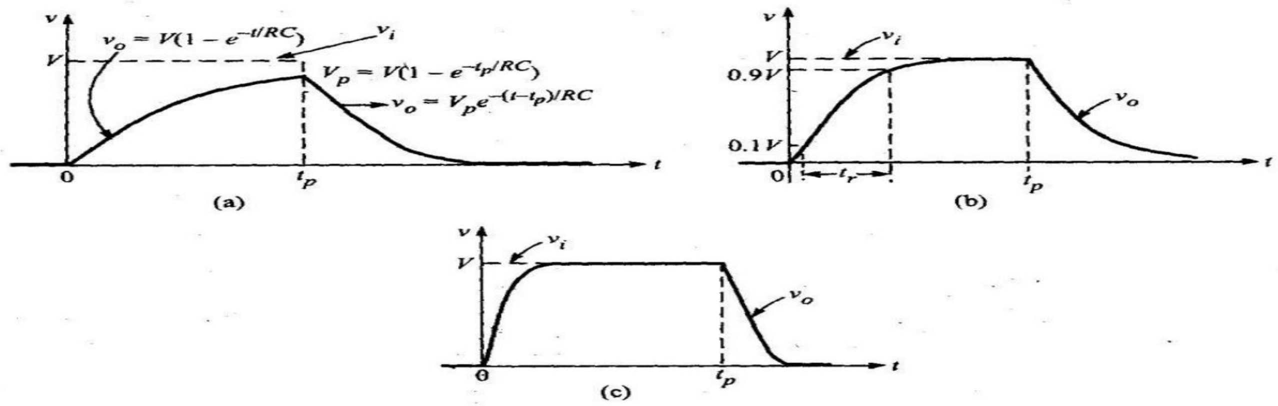


Figure 1.5 Pulse response for (a) $RC >> t_p$, (b) $RC < t_p$, and (c) $RC << t_p$.

If the time constant *RC* of the circuit is very small, the capacitor charges and discharges very quickly and the rise time *tr* will be small and so the distortion in the wave shape is small.

For minimum distortion (i.e. for preservation of wave shape), the rise time must be small compared to the pulse width *tp.* If the upper 3-dB frequency f2 is chosen equal to the reciprocal of the pulse width *tp,* i.e. if f2 = 1/tp then *tr* = 0.35tp and the output is as shown in Figure 1.5(b), which for many applications is a reasonable reproduction of the input. As a rule of thumb, we can say:

A pulse shape will be preserved if the 3-dB frequency is approximately equal to the reciprocal of the pulse width.

Thus to pass a 0.25 μ.s pulse reasonably well requires a circuit with an upper cut-off frequency of the order of 4 MHz.

Square wave input

A square wave is a periodic waveform which maintains itself at one constant level *V' with respect to ground for a time T1 and then changes abruptly to another level V", and remains constant at that level for a time T2, and repeats itself at regular intervals of T = T1 + T2. A square* wave may be treated as a series of positive and negative steps. The shape of the output waveform for a square wave input depends on the time constant of the circuit.

Square-Wave Input

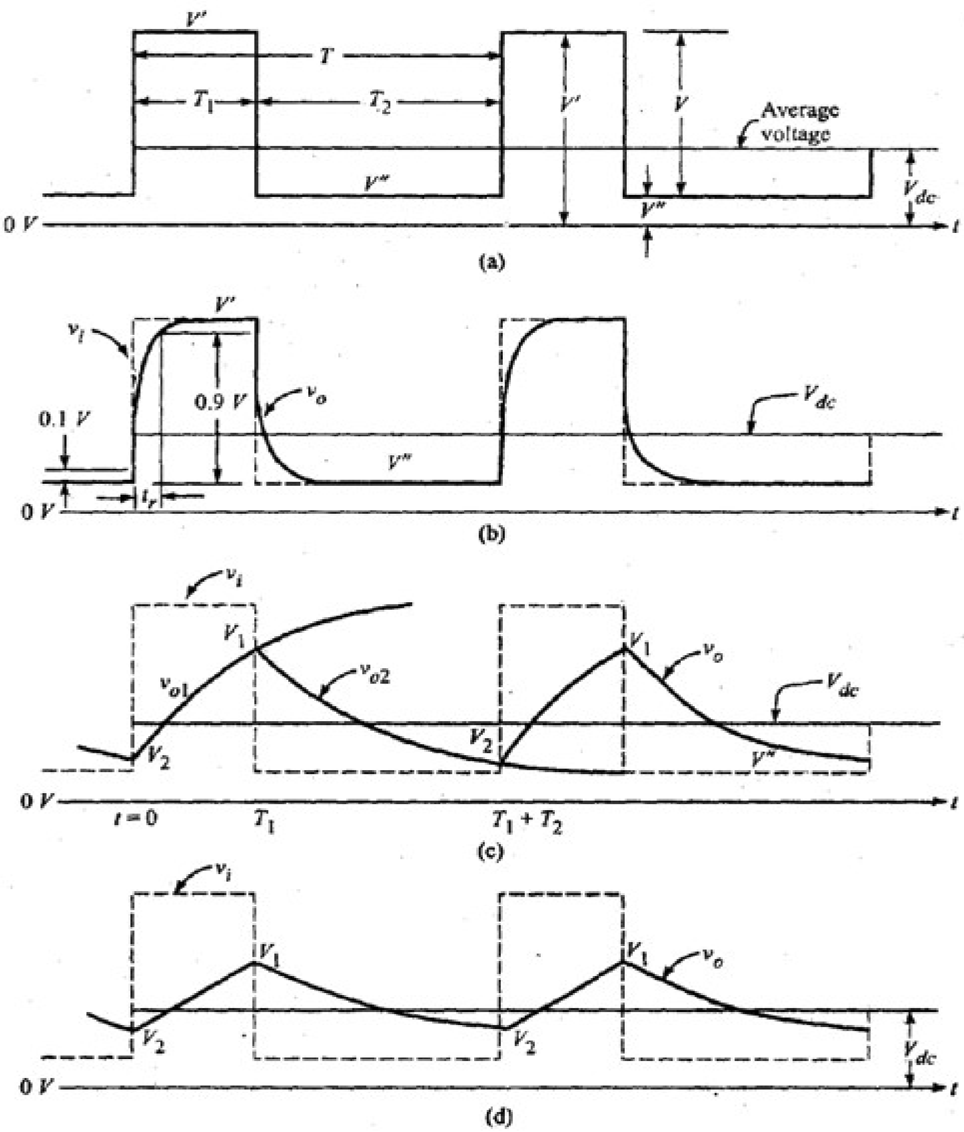


Figure 1.6 Response of a low-pass *RC* circuit to a square wave input: (a) square-wave input wave form, (b) output waveform for $RC \ll T$, (c) output waveform for $RC \approx T$, and (d) output waveform for $RC \gg T$.

In Figure 1.6(c), the equation for the rising portion is

$$v_{01} = V' - (V' - V_2)e^{-t/RC}$$

where V_2 is the voltage across the capacitor at $t = 0$, and V' is the level to which the capacitor can charge.

The equation for the falling portion is

$$v_{02} = V'' - (V'' - V_1)e^{-(t - T_1)/RC}$$

where V_1 is the voltage across the capacitor at $t = T_1$ and V'' is the level to which the capacitor can discharge.

Setting $v_{01} = V_1$ at $t = T_1$,

$$V_1 = V' - (V' - V_2)e^{-T_1/RC} = V'(1 - e^{-T_1/RC}) + V_2e^{-T_1/RC}$$

Setting $v_{02} = V_2$ at $t = T_1 + T_2$,

$$V_2 = V'' - (V'' - V_1)e^{-(T_1+T_2-T_1)/RC} = V''(1 - e^{-T_2/RC}) + V_1e^{-T_2/RC}$$

Substituting this value of V_2 in the expression for V_1,

$$V_1 = V'(1 - e^{-T_1/RC}) + [V''(1 - e^{-T_2/RC}) + V_1e^{-T_2/RC}]e^{-T_1/RC}$$

i.e.

$$V_1 = \frac{V'(1-e^{-T_1/RC})+V''(1-e^{-T_2/RC})e^{-T_1/RC}}{1-e^{-(T_1+T_2)/RC}}$$

Similarly substituting the value of V_1 in the expression for V_2,

$$V_2 = \frac{V''(1 - e^{-T_2/RC}) + V'(1 - e^{-T_1/RC})\,e^{-T_2/RC}}{1 - e^{-(T_1+T_2)/RC}}$$

For a symmetrical square wave with zero average value,

$$T_1 = T_2 = \frac{T}{2} \text{ and } V' = -V'' = \frac{V}{2}. \text{ So, } V_2 \text{ will be equal to } -V_1$$

$$\therefore \qquad V_1 = \frac{\frac{V}{2}(1 - e^{-T/2RC}) - \frac{V}{2}(1 - e^{-T/2RC})\, e^{-T/2RC}}{1 - e^{-T/RC}}$$

$$= \frac{V}{2} \frac{1 - e^{-T/2RC} - e^{-T/2RC} + e^{-T/RC}}{1 - e^{-T/RC}}$$

$$= \frac{V}{2} \frac{(1 - e^{-T/2RC})^2}{(1 + e^{-T/2RC})(1 - e^{-T/2RC})}$$

$$= \frac{V}{2} \left(\frac{1 - e^{-T/2RC}}{1 + e^{-T/2RC}} \right)$$

$$= \frac{V}{2} \left(\frac{e^{T/2RC} - 1}{e^{T/2RC} + 1} \right)$$

$$= \frac{V}{2} \left(\frac{e^{2x} - 1}{e^{2x} + 1} \right) = \frac{V}{2} \tanh x$$

where $x = \frac{T}{4RC}$ and T is the period of the square wave.

Now, $$V_2 = -V_1 = -\frac{V}{2}\left(\frac{1 - e^{-T/2RC}}{1 + e^{-T/2RC}} \right) = \frac{V}{2} \left(\frac{1 - e^{T/2RC}}{1 + e^{T/2RC}} \right)$$

1.1.5 Ramp Input

When a low-pass RC circuit shown in Figure 1.1 is excited by a ramp input, i.e.

$$v_i(t) = \alpha t, \text{ where } \alpha \text{ is the slope of the ramp}$$

we have,

$$V_i(s) = \frac{\alpha}{s^2}$$

From the frequency domain circuit of Figure 1.2(a), the output is given by

$$V_o(s) = V_i(s)\,\frac{\dfrac{1}{Cs}}{R + \dfrac{1}{Cs}} = \frac{\alpha}{s^2}\cdot\frac{1}{1 + RCs} = \frac{\alpha}{RC}\,\frac{1}{s^2\left(s + \dfrac{1}{RC}\right)}$$

Using Partial Fractions

$$= \frac{\alpha}{RC}\left[\frac{-(RC)^2}{s} + \frac{RC}{s^2} + \frac{(RC)^2}{s + \dfrac{1}{RC}}\right]$$

i.e. $$V_o(s) = \frac{-\alpha RC}{s} + \frac{\alpha}{s^2} + \frac{\alpha RC}{s + \dfrac{1}{RC}}$$

Taking the inverse Laplace transform on both sides,

$$v_o(t) = -\alpha RC + \alpha t + \alpha RC e^{-t/RC}$$
$$= \alpha(t - RC) + \alpha RC e^{-t/RC}$$

If the time constant RC is very small, $e^{-t/RC} \approx 0$

$$\therefore \quad v_o(t) = \alpha(t - RC)$$

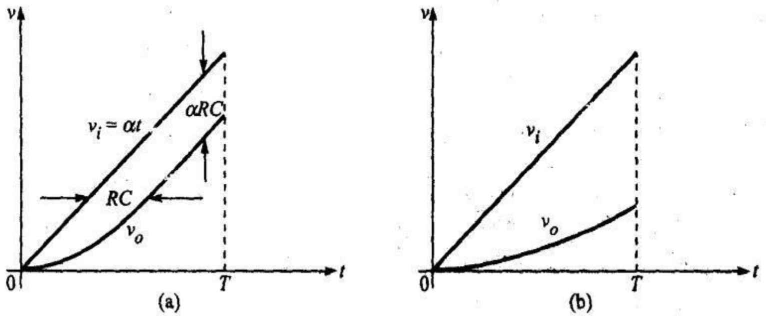


Figure 1.7 Response of a low-pass *RC* circuit for a ramp input for (a) *RC/T* << 1 and (b) *RC/T* >> 1.

Expanding $e^{-t/RC}$ in to an infinite series in t/RC in the above equation for $v_o(t)$,

$$v_o(t) = \alpha(t - RC) + \alpha RC\left(1 - \frac{t}{RC} + \left(\frac{t}{RC}\right)^2 \frac{1}{2!} - \left(\frac{t}{RC}\right)^3 \frac{1}{3!} + \dots\right)$$

$$= \alpha t - \alpha RC + \alpha RC - \alpha t + \frac{\alpha t^2}{2RC} - \dots$$

$$\approx \frac{\alpha t^2}{2RC} \approx \frac{\alpha}{RC}\left(\frac{t^2}{2}\right)$$

This shows that a quadratic response is obtained for a linear input and hence the circuit acts as an integrator for $RC/T \gg 1$.

The transmission error e_t for a ramp input is defined as the difference between the input and the output divided by the input at the end of the ramp, i.e. at $t = T$.

For $RC/T \ll 1$,

$$e_t = \left.\frac{\alpha t - (\alpha t - \alpha RC)}{\alpha t}\right|_{t=T}$$

$$= \frac{\alpha RC}{\alpha T} = \frac{RC}{T} = \frac{1}{2\pi f_2 T}$$

THE LOW-PASS *RC CIRCUIT AS AN INTEGRATOR*

If the time constant of an *RC low-pass circuit is very large, the capacitor charges very slowly and so almost all the input voltage appears across the resistor for small values of time.*
Then, the current in the circuit is vi(t)lR and the output signal across C is

$$v_o(t) = \frac{1}{C}\int i(t)\,dt = \frac{1}{C}\int \frac{v_i(t)}{R}\,dt = \frac{1}{RC}\int v_i(t)\,dt$$

Hence the output is the integral of the input, i.e. if $v_i(t) = \alpha t$; then

$$v_o(t) = \frac{\alpha t^2}{2RC}$$

As time increases, the voltage drop across *C does not remain negligible compared with that across R and the output will not remain the integral of the input.*

If the time constant of an RC low-pass circuit is very large in comparison with the time required for the input signal to make an appreciable change, the circuit acts as an integrator.

A criterion for good integration in terms of steady-state analysis is as follows: The low-pass circuit acts as an integrator provided the time constant of the circuit *RC > 15T, where T is the period of the input sine wave.*

An *RC integrator converts a square wave into a triangular wave. Integrators are almost invariably preferred over differentiators in analog computer applications for the following reasons:*

1. It is easier to stabilize an integrator than a differentiator because the gain of an integrator decreases with frequency whereas the gain of a differentiator increases with frequency.
2. An integrator is less sensitive to noise voltages than a differentiator because of its limited bandwidth.
3. The amplifier of a differentiator may overload if the input waveform changes very rapidly.
4. It is more convenient to introduce initial conditions in an integrator.

THE HIGH-PASS *RC CIRCUIT*

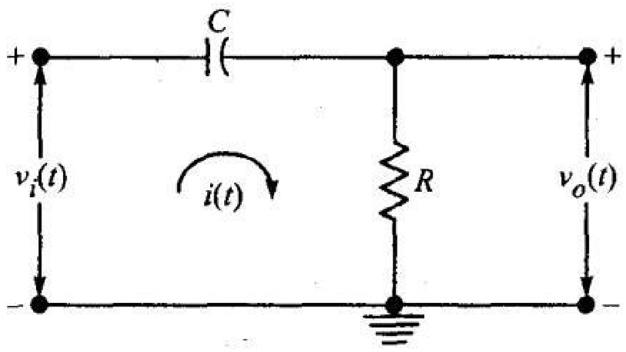


Figure 1.30 The high-pass *RC* circuit.

At zero frequency the reactance of the capacitor is infinity and so it blocks the input and hence the output is zero.

This capacitor is called the *blocking capacitor and this circuit, also called the capacitive coupling circuit, is used to provide dc isolation between the input and the output.*

This circuit attenuates low-frequency signals and allows transmission of high-frequency signals with little or no attenuation, it is called a high-pass circuit.

Sinusoidal Input

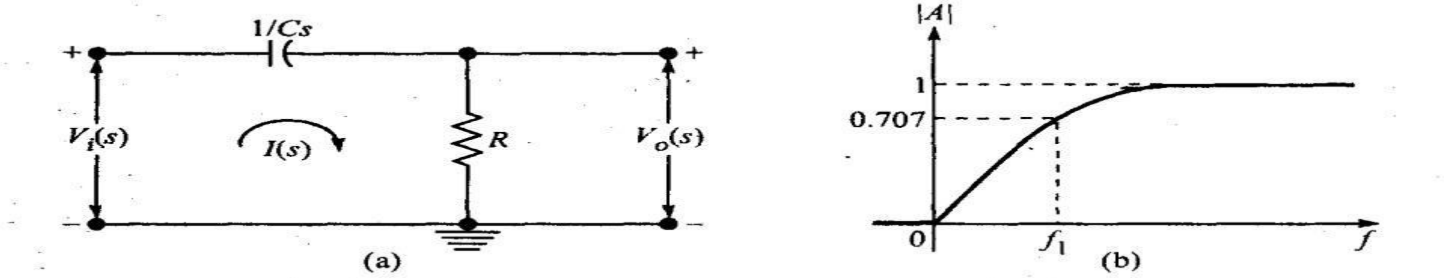


Figure 1.31 (a) Laplace transformed high-pass circuit and (b) gain versus frequency plot.

For a sinusoidal input v i , the output signal *v0 increases in amplitude with increasing frequency.*
The frequency at which the gain is 1/V2 of its maximum value is called the lower cut-off or lower 3-dB frequency.

For a high-pass circuit, there is no upper cut-off frequency because all high frequency signals are transmitted with zero attenuation.
Therefore, f2 – f1. Hence bandwidth B.W= f2 - fi =∞

Expression for the lower cut-off frequency

For the high-pass *RC circuit shown in Figure 1.31 (a), the magnitude of the steady-state gain A, and the angle ϑ by which the output leads the input are given by*

$$A = \frac{V_o(s)}{V_i(s)} = \frac{R}{R + \frac{1}{Cs}} = \frac{1}{1 + \frac{1}{RCs}}$$

Putting $s = j\omega$; $A = \dfrac{1}{1 - j\dfrac{1}{\omega RC}} = \dfrac{1}{1 - j\dfrac{1}{2\pi fRC}}$

$\therefore$ $|A| = \dfrac{1}{\sqrt{1 + \left(\dfrac{1}{2\pi fRC}\right)^2}}$ and $\theta = -\tan^{-1}\dfrac{1}{2\pi fRC}$

At the lower cut-off frequency f_1, $|A| = 1/\sqrt{2}$

$$\therefore \quad \frac{1}{\sqrt{1 + \left(\frac{1}{2\pi f_1 RC}\right)^2}} = \frac{1}{\sqrt{2}}$$

Squaring and equating the denominators,

$$\frac{1}{2\pi f_1 RC} = 1 \quad \text{i.e.} \quad f_1 = \frac{1}{2\pi RC}$$

This is the expression for the lower cut-off frequency of a high-pass circuit.

Relation between f1 and tilt

The lower cut-off frequency of a high-pass circuit is $f_1 = 1 / 2\pi RC$. *The lower cut-off frequency produces a tilt. For a 10% change in capacitor voltage, the time or pulse width involved is*

$$t = 0.1RC = \text{PW}$$

$$\therefore \quad \frac{\text{PW}}{RC} = 0.1 = \text{Fractional tilt}$$

$$\therefore \quad \text{Fractional tilt} = \frac{\text{PW}}{RC} = 2\pi f_1 \cdot \text{PW}$$

This equation applies only when the tilt is 10% or less.

Step Input

When a step signal of amplitude *V volts shown in Figure 1.32(a) is applied to the high-pass RC circuit of Figure 1.30, since the voltage across the capacitor cannot change instantaneously the output will be just equal to the input at t = 0 (for t < 0,* $v_i = 0$ *and* $v_0 = 0$*).*

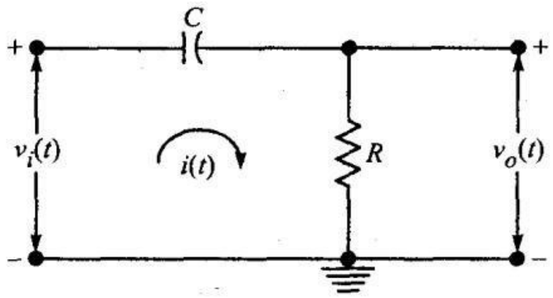


Figure 1.30 The high-pass *RC* circuit.

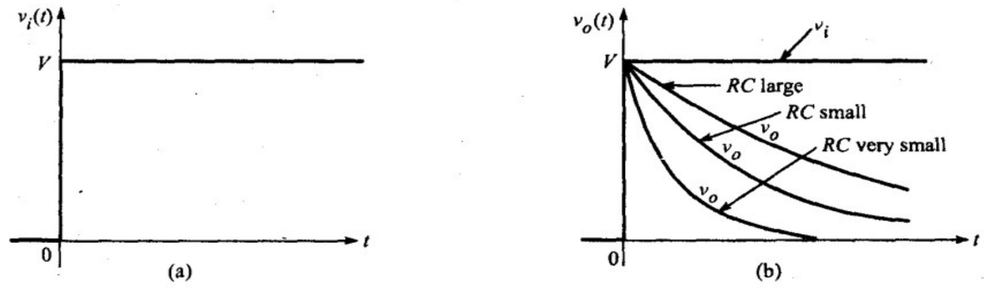


Figure 1.32 (a) Step input and (b) step response for different time constants.

Later when the capacitor charges exponentially, the output reduces exponentially with the same time constant *RC. The expression for the output voltage for t > 0 is given by*

$$v_o(t) = v_f - (v_f - v_i)e^{-t/RC}$$
$$= 0 - (0 - V)e^{-\frac{t}{RC}} = V\,e^{-t/RC}$$

V_{final} is zero and $V_{initial}$ is V for RC high pass circuit

Figure 1.32(b) shows the response of the circuit for large, small, and very small time constants. For *t > 5r, the output will reach more than 99% of its final value. Hence although the steady state is approached asymptotically, for most applications we may assume that the final value has been reached after 5f. If the initial slope of the exponential is maintained, the output falls to zero in a time t = T.*

Pulse Input

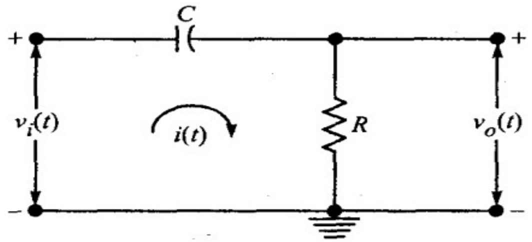


Figure 1.30 The high-pass *RC* circuit.

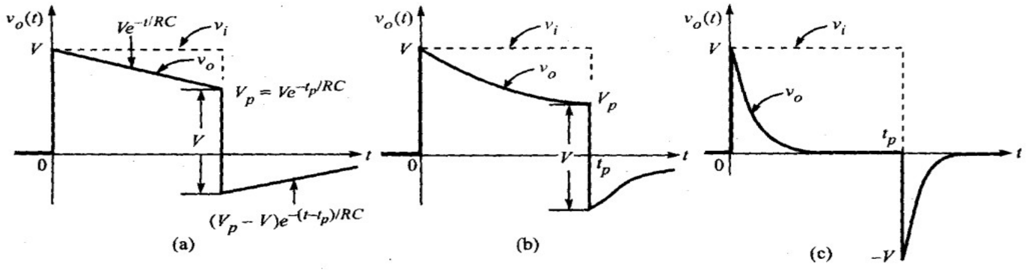


Figure 1.33 Pulse response for (a) $RC >> t_p$, (b) RC comparable to t_p, and (c) $RC << t_p$.

A pulse of amplitude *V and duration tp shown in Figure 1.4(a) is nothing but the sum of a positive step of amplitude V starting at t = 0 and a negative step of amplitude V starting at tp as shown in above Figure*

Square-Wave Input

A square wave shown in Figure 1.34(a) is a periodic waveform, which maintains itself at one constant level *V with respect to ground for a time T1 and then changes abruptly to another level V" and remains constant at that level for a time T2, and then repeats itself at regular intervals of T = T1 + T2. A square wave may be treated as a series of positive and negative steps.*

The shape of the output depends on the time constant of the circuit. Figures 1.34(b), 1.34(c), 1.34(d), and 1.34(e) show the output waveforms of the high-pass *RC circuit under .steady-state conditions for the cases (a) RC » T, (b) RC > T, (c) RC - T, and (d) RC « T respectively.*

When the time constant is arbitrarily large (i.e. *RC/T1 and RC/T2 are very very large in comparison to unity) the output is same as the input but with zero dc level. When RC > T, the output is in the form of a tilt. When RC is comparable to T, the output rises and falls exponentially.*

When *RC « T (i.e. RCIT\ and RC/T2 are very small in comparison to unity), the output consists of alternate positive and negative spikes.*

for any periodic input waveform under steady-state conditions, the average level of the output waveform from the high-pass circuit of Figure 1.30 is always zero independently of the dc level of the input. The proof is as follows:

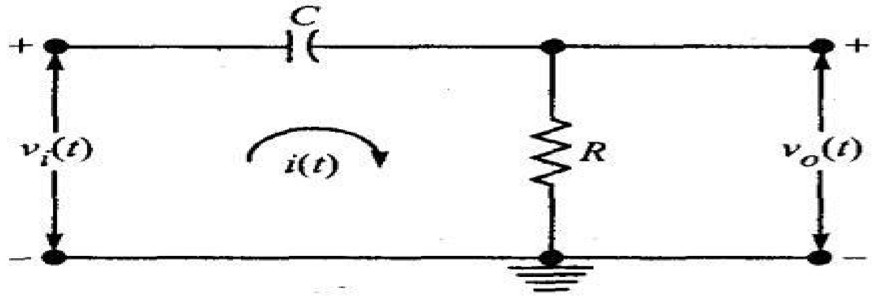


Figure 1.30 The high-pass RC circuit.

$$v_i(t) = \frac{1}{C}\int i(t)\,dt + v_o(t)$$

$$= \frac{1}{RC}\int v_o(t)\,dt + v_o(t) \qquad \left(\because\ i(t) = \frac{v_o(t)}{R}\right)$$

Differentiating,

$$\frac{dv_i(t)}{dt} = \frac{v_o(t)}{RC} + \frac{dv_o(t)}{dt}$$

Multiplying by dt and integrating this equation over one period T,

$$\int_{t=0}^{t=T} dv_i(t) = \int_{t=0}^{t=T} \frac{v_o(t)\,dt}{RC} + \int_{t=0}^{t=T} dv_o(t)$$

i.e. $$v_i(T) - v_i(0) = \frac{1}{RC}\int_0^T v_o(t)\,dt + v_o(T) - v_o(0)$$

Under steady-state conditions, the output waveform (as well as the input signal) is repetitive with a period T so that $v_o(T) = v_o(0)$ and $v_i(T) = v_i(0)$.

Under steady-state conditions, the output waveform (as well as the input signal) is repetitive with a period *T so that v0(T) = v0(0) and vi T) = vi(0).*

Hence

$$\int_0^T v_o(t)\,dt = 0.$$

%Tilt: %Tilt is defined as decay in the amplitude of the output voltage wave due to the input voltage maintaining constant level.

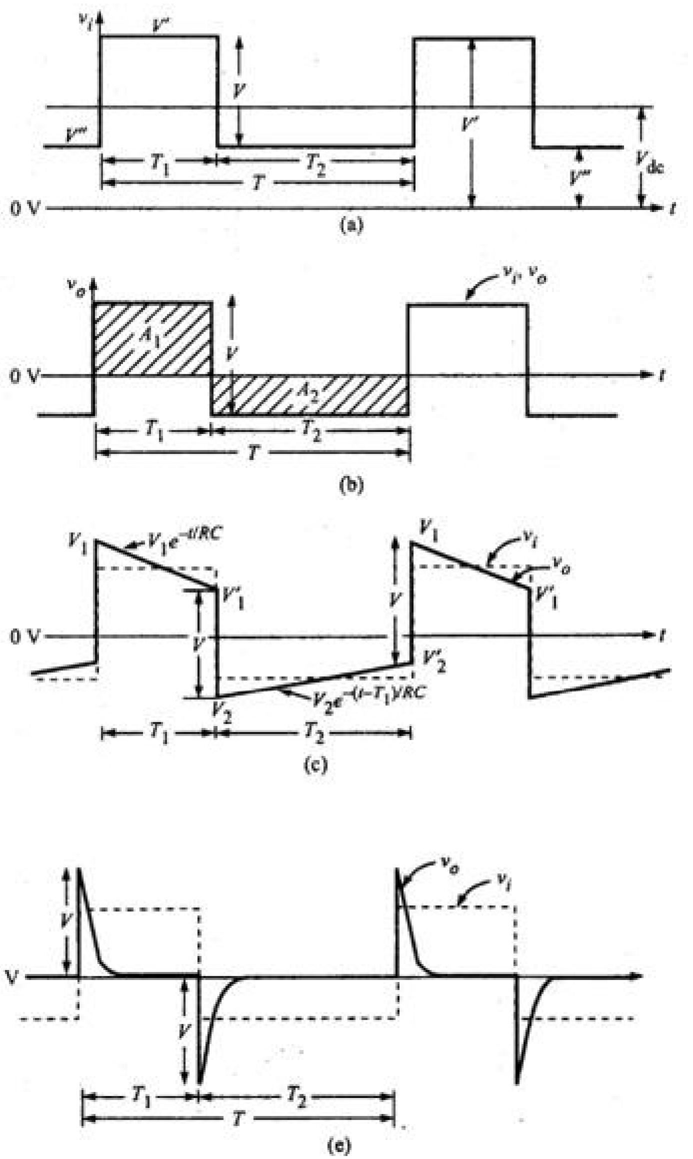


Figure 1.34 (a) A square wave input, (b) output when RC is arbitrarily large, (c) output when $RC > T$, (d) output when RC is comparable to T, and (e) output when $RC << T$.

Under steady-state conditions, the capacitor charges and discharges to the same voltage levels in each cycle. So the shape of the output waveform is fixed.

For $0 < t < T_1$, the output is given by $v_{o1} = V_1 e^{-t/RC}$

At $t = T_1$, $v_{o1} = V_1' = V_1 e^{-T_1/RC}$

For $T_1 < t < T_1 + T_2$, the output is $v_{o2} = V_2 e^{-(t-T_1)/RC}$

At $t = T_1 + T_2$, $v_{o2} = V_2' = V_2 e^{-T_2/RC}$

Also $V_1' - V_2 = V$ and $V_1 - V_2' = V$

From these relations V_1, V_1', V_2 and V_2' can be computed.

Expression for the percentage tilt

%Tilt: %Tilt is defined as decay in the amplitude of the output voltage wave due to the input voltage maintaining constant level.

We will derive an expression for the percentage tilt when the time constant *RC of the circuit is very large compared to the period of the input waveform, i.e. RC » T. For a symmetrical square wave with zero average value*

$$V_1 = -V_2,\ \text{i.e. } V_1 = |V_2|,\ V_1' = -V_2',\ \text{i.e. } V_1' = |V_2'|,\ \text{and } T_1 = T_2 = \frac{T}{2}$$

The output waveform for $RC >> T$ is shown in Figure 1.35. Here,

$$V_1' = V_1 e^{-T/2RC} \quad \text{and} \quad V_2' = V_2 e^{-T/2RC}$$

$$V_1 - V_2' = V$$

i.e.

$$V_1 - V_2 e^{-T/2RC} = V_1 + V_1 e^{-T/2RC} = V$$

$\therefore$

$$V_1 = \frac{V}{1 + e^{-T/2RC}} \quad \text{or} \quad V = V_1 (1 + e^{-T/2RC})$$

$$\%\ \text{tilt},\ P = \frac{V_1 - V_1'}{\frac{V}{2}} \times 100\% = \frac{V_1 - V_1 e^{-T/2RC}}{V_1 (1 + e^{-T/2RC})} \times 200\% = \frac{1 - e^{-T/2RC}}{1 + e^{-T/2RC}} \times 200\%$$

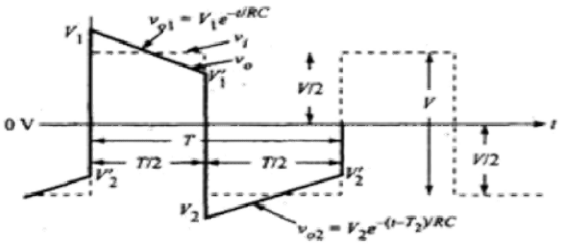


Figure 1.35 Linear tilt of a symmetrical square wave when $RC >> T$.

When the time constant is very large, i.e. $\frac{T}{RC} << 1$

$$P = \frac{1 - \left[1 + \left(\frac{-T}{2RC}\right) + \left(\frac{-T}{2RC}\right)^2 \frac{1}{2!} + \ldots\right]}{1 + 1 + \left(\frac{-T}{2RC}\right) + \left(\frac{-T}{2RC}\right)^2 \frac{1}{2!} + \ldots} \times 200\%$$

$$= \frac{\frac{T}{2RC}}{2} \times 200\%$$

$$= \frac{T}{2RC} \times 100\%$$

$$= \frac{\pi f_1}{f} \times 100\%$$

where $f_1 = \frac{1}{2\pi RC}$ is the lower cut-off frequency of the high-pass circuit.

Ramp Input

A waveform which is zero for *t < 0 and which increases linearly with time for t > 0 is called a ramp or sweep voltage.*

When the high-pass *RC circuit is excited by a ramp input v i (t) = at, where a is the slope of the ramp, then*

$$V_i(s) = \frac{\alpha}{s^2}$$

From the Laplace transformed circuit of Figure 1.31(a),

$$V_o(s) = V_i(s)\frac{R}{R + \frac{1}{Cs}} = \frac{\alpha}{s^2}\frac{RCs}{1 + RCs}$$

$$= \frac{\alpha}{s\left(s + \frac{1}{RC}\right)} = \alpha RC\left(\frac{1}{s} - \frac{1}{s + \frac{1}{RC}}\right)$$

Taking the inverse Laplace transform on both sides,

$$v_o(t) = \alpha RC(1 - e^{-t/RC})$$

For times t which are very small in comparison with RC, we have

$$v_o(t) = \alpha RC\left[1-\left\{1+\left(\frac{-t}{RC}\right)+\left(\frac{-t}{RC}\right)^2\frac{1}{2!}+\left(\frac{-t}{RC}\right)^3\frac{1}{3!}+\ldots\right\}\right]$$

$$= \alpha RC\left[\frac{t}{RC}-\frac{t^2}{2(RC)^2}+\ldots\right]$$

$$= \alpha t-\frac{\alpha t^2}{2RC}=\alpha t\left(1-\frac{t}{2RC}\right)$$

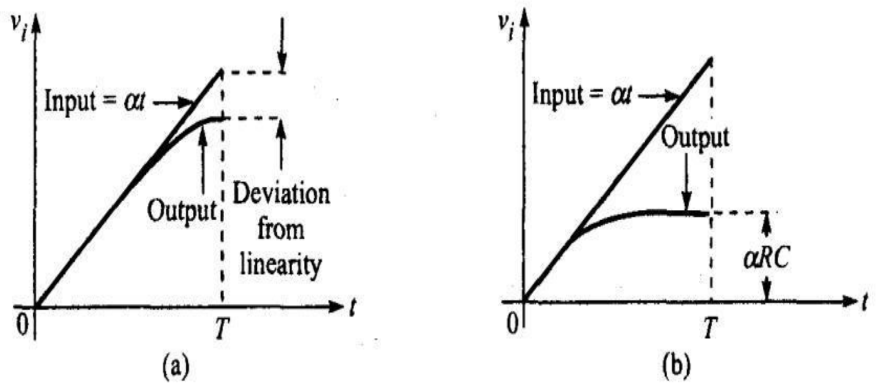


Figure 1.36 Response of the high-pass circuit for a ramp input when (a) $RC >> T$ and (b) $RC << T$.

Transmission error:

$$e_t = \left.\frac{v_i - v_o}{v_i}\right|_{t=T} \approx \left.\frac{\alpha t - \alpha t\left(1 - \frac{t}{2RC}\right)}{\alpha t}\right|_{t=T} \approx \frac{T}{2RC} = \pi f_1 T$$

where $f_1 = \frac{1}{2\pi RC}$ is the lower 3-dB frequency of the high-pass circuit.

THE HIGH-PASS *RC CIRCUIT AS A DIFFERENTIATOR*

When the time constant of the high-pass RC circuit is very very small, the capacitor charges very quickly; so almost all the input v,(0 appears across the capacitor and the voltage across the resistor will be negligible compared to the voltage across the capacitor. Hence the current is determined entirely by the capacitence.

Then the current

$$i(t) = C\frac{dv_i(t)}{dt}$$

and the output signal across R is

$$v_o(t) = RC\frac{dv_i(t)}{dt}$$

Thus we see that the output is proportional to the derivative of the input

The high-pass RC circuit acts as a differentiator provided the RC time, constant of the circuit is very small in comparison with the time required for the input signal to make an appreciable change.

The derivative of a step signal is an impulse of infinite amplitude at the occurrence of the discontinuity of step.

The derivative of an ideal pulse is a positive impulse followed by a delayed negative impulse, each of infinite amplitude and occurring at the points of discontinuity.

The derivative of a square wave is a waveform which is uniformly zero except, at the points of discontinuity.

MULTIVIBRATORS

Multi means many vibrator means oscillator. A circuit which can oscillate at a number of frequencies is called a *multivibrator. Basically there are three types of multivibrators:*

1. Bistable multivibrator – consists of two stable states
2. Monostable multivibrator - consists of one stable states and one quasi stable states
3. Astable multivibrator- consists of two quasi stable states or No stable states

A bistable multivibrator has got two stable states, a monostable multivibrator has got only one stable state (the other state being quasi stable) and the astable multivibrator has got no stable state (both the states being quasi stable).

The stable state of a multivibrator is the state in which the device can stay permanently. Only when a proper external triggering signal is applied, it will change its state. Quasi stable state means temporarily stable state. The device cannot stay permanently in this state.

After a predetermined time, the device will automatically come out of the quasi stable state.

A bistable multivibrator is the basic memory element. It is used to perform many digital operations such as counting and storing of binary data. It also finds extensive applications in the generation and processing of pulse type waveforms.

The monostable multivibrator finds extensive applications in pulse circuits. Mostly it is used as a gating circuit or a delay circuit.

The astable circuit is used as a master oscillator to generate square waves. It is often a basic source of fast waveforms. It is a free running oscillator. It is called a *square wave generator. It is also termed a relaxation oscillator.*

BISTABLE MULTIVIBRATOR

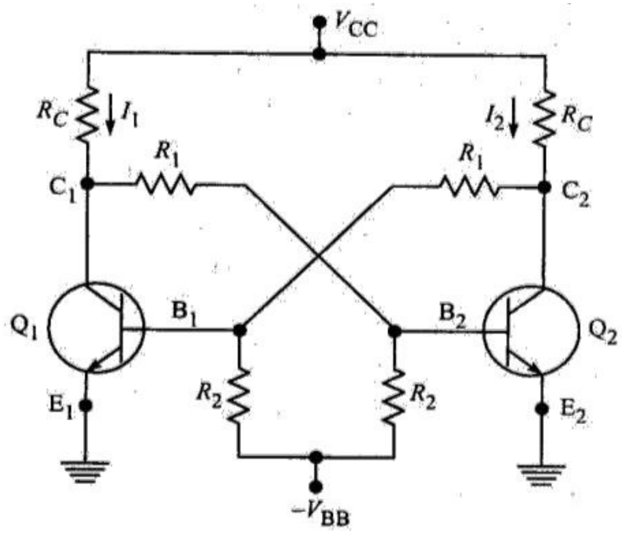
A bistable multivibrator is a multivibrator which can exist indefinitely in either of its two stable states and which can be induced to make an abrupt transition from one state to the other by means of external excitation. In a bistable multivibrator both the coupling elements are resistors (dc coupling). The bistable multivibrator is also called a multi, Eccles-Jordan circuit (after its inventors), trigger circuit, scale-of-two toggle circuit, flip-flop, and binary. There are two types of bistable multivibrators:

1. Collector coupled bistable multivibrator
2. Emitter coupled bistable multivibrator

There are two types of collector-coupled bistable multivibrators:

1. Fixed-bias bistable multivibrator
2. Self-bias bistable multivibrator

A FIXED-BIAS BISTABLE MULTIVIBRATOR



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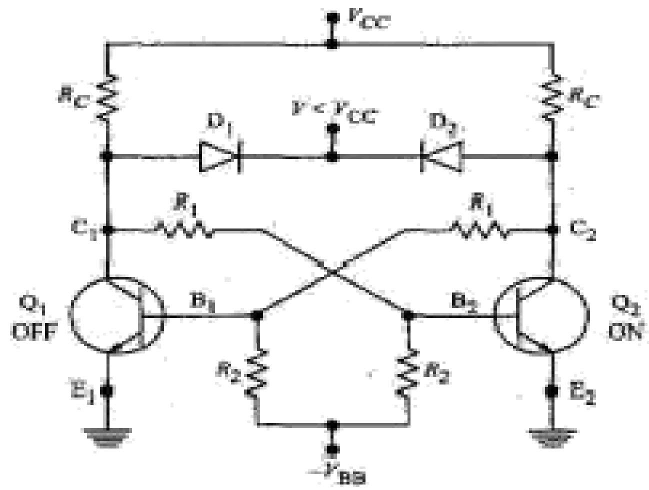


Fixed bias binary with collector catching diodes

Standard specifications

In the cut-off region, i.e. for the OFF state

V_{BE} (cut-off) : ≤ 0 V for silicon transistor
≤ -0.1 V for germanium transistor

In the saturation region, i.e. for the ON state

V_{BE} (sat) : 0.7 V for silicon transistor
0.3 V for germanium transistor
V_{CE} (sat) : 0.3 V for silicon transistor
0.1 V for germanium transistor

The above values hold good for n-p-n transistors. For p-n-p transistors the above values with opposite sign are to be taken.

Test for saturation

To test whether a transistor is really in saturation or not evaluate the collector current i_C and the base current i_B independently.

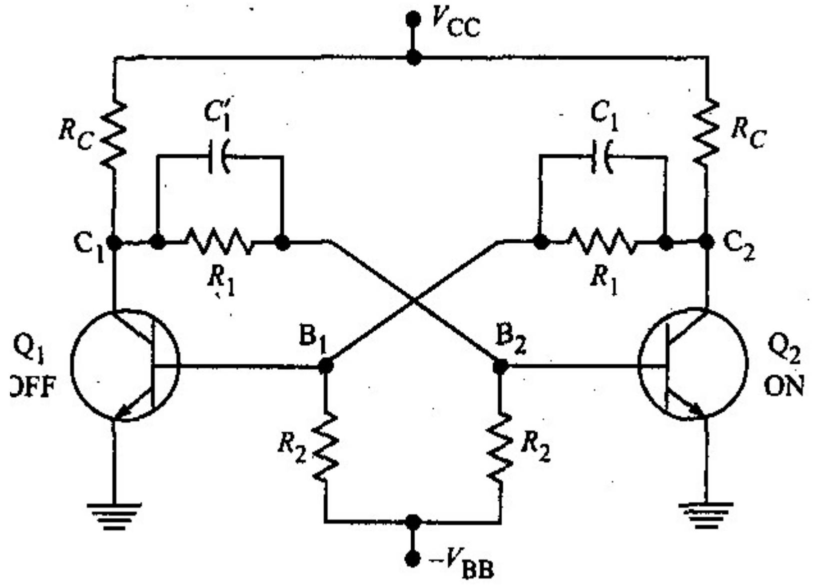
If $i_B > i_B$ (min), where i_B (min) = i_C/h_{FE} (min) the transistor is really in saturation.
If $i_B \leq i_B$ (min), the transistor is not in saturation.

Test for cut-off

To test whether a transistor is really cut-off or not, find its base-to-emitter voltage. If V_{BE} is negative for an n-p-n transistor or positive for a p-n-p transistor, the transistor is really cut-off.

COMMUTATING CAPACITORS

We know that the bistable multivibrator has got two stable states and that it can remain in either of its two stable states indefinitely. It can change state only when a triggering signal such as a pulse from some external source is applied. When a triggering signal is applied, conduction has to transfer from one device to another. The transition time is defined as the interval during which conduction transfers from one transistor to another.



The reason for this transition time is—even though the input signal at the base of a transistor may be transferred to the collector with zero rise time, the signal at the collector of the transistor cannot be transferred to the base of the other transistor instantaneously.

This is because the input capacitance C_i present at the base of the transistor makes the *R\-R2 attenuator act as an uncompensated attenuator and so it will have a finite rise time, tr = (R1//R2)Ci. The transition time may be reduced by compensating this attenuator by introducing a small capacitor in parallel with the coupling resistors R1and R2 of the binary as shown in Figure 4.21.*

THE EMITTER-COUPLED BINARY (THE SCHMITT TRIGGER CIRCUIT)

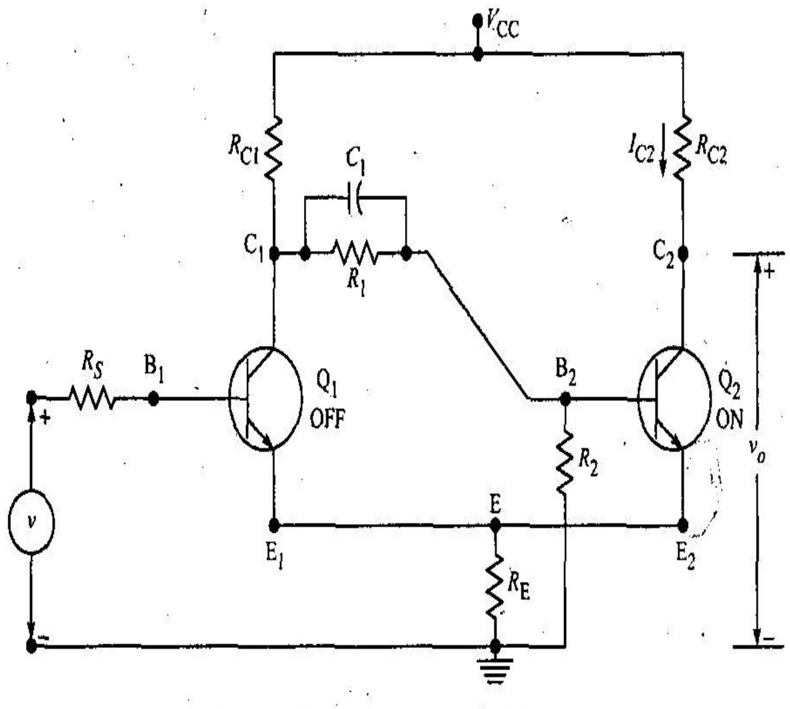
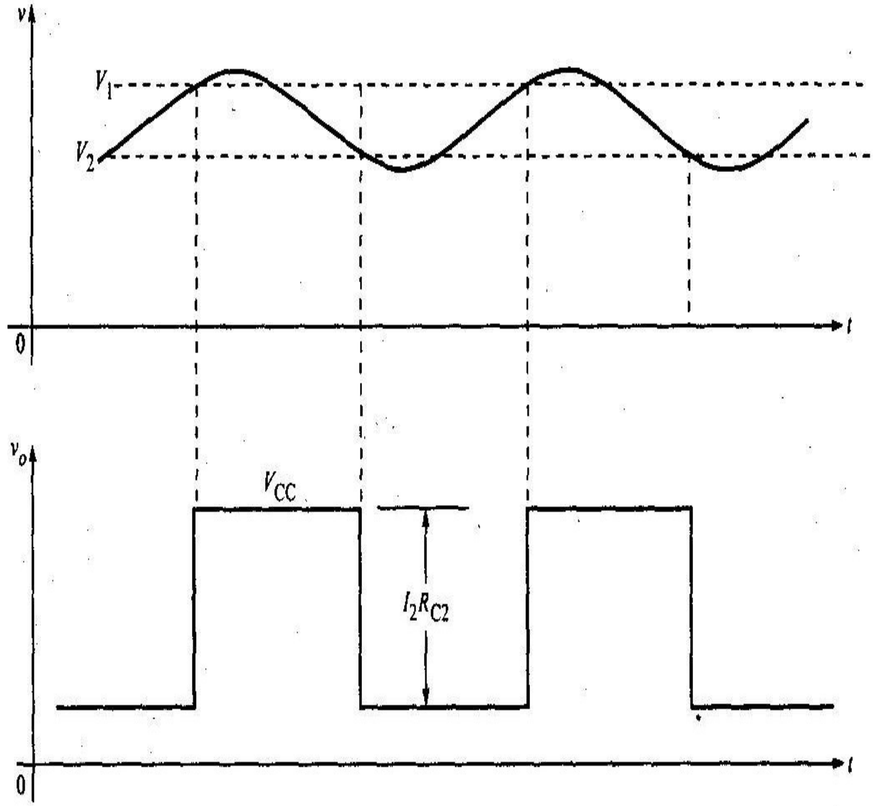


Figure 4.29 An emitter-coupled binary.



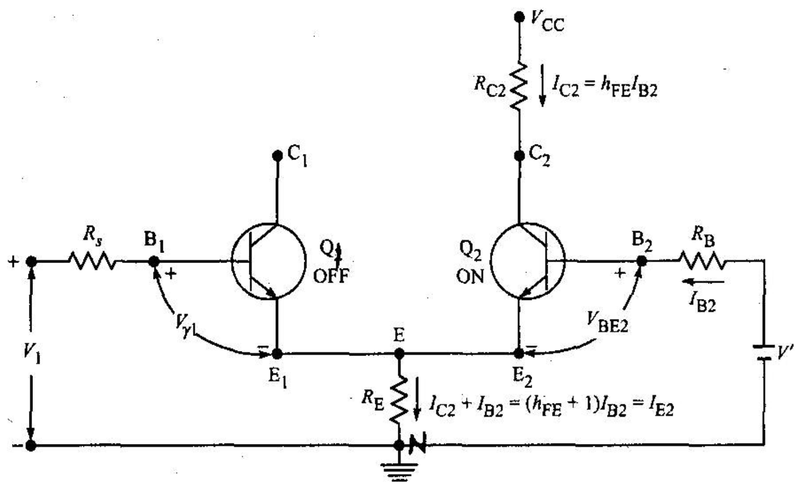
Derivation of expression for UTP

The upper triggering point UTP is defined as the input voltage V_1 *at which the transistor* Q_i *just enters* into conduction.

To calculate V_b , we have to first find the current in Q2 when Q1 just enters into conduction. For this we have to find the Thevenin's equivalent voltage *V and the Thevenin's equivalent* resistance tfB at the base of Q2, where

$$V' = V_{CC}\frac{R_2}{R_2 + R_{C1} + R_1} \quad \text{and} \quad R_B = R_2 \parallel (R_{C1} + R_1) = \frac{R_2(R_{C1} + R_1)}{R_2 + R_{C1} + R_1}$$

It is possible for Q2 to be in its active region or to be in saturation. Assuming that Q2 is in its active region



Writing KVL around the base loop of Q2,

$$V' - I_{B2}R_B - V_{BE2} - I_{B2}(h_{FE} + 1)R_E = 0$$

$\therefore$
$$I_{B2} = \frac{V' - V_{BE2}}{(h_{FE} + 1)\,R_E + R_B}$$

Hence
$$V_{EN} = I_{B2}(h_{FE} + 1)\ R_E = \frac{(V' - V_{BE2})\,(h_{FE} + 1)\,R_E}{R_B + R_E\,(h_{FE} + 1)}$$

Also
$$V_{EN1} = V_{EN} = V_{EN2}$$

Since Q_1 is just at cut-in, $I_{B1} = 0$ and $V_{BE1} = V_{\gamma 1}$

$\therefore$
$$V_1 = V_{EN1} + V_{BE1} + I_{B1}R_S = V_{EN} + V_{\gamma 1}$$

If $R_E(h_{FE} + 1) >> R_B$, the drop across R_B may be neglected compared to the drop across R_E.

$\therefore$
$$V_{EN} = V' - V_{BE2}$$

and
$$V_1 = V' - V_{BE2} + V_{\gamma 1}$$

Since VyJ is the voltage from base to emitter at cut-in where the loop gain just exceeds unity, it differs from VBE2 in the active region by only 0.1 V for either Ge or Si.

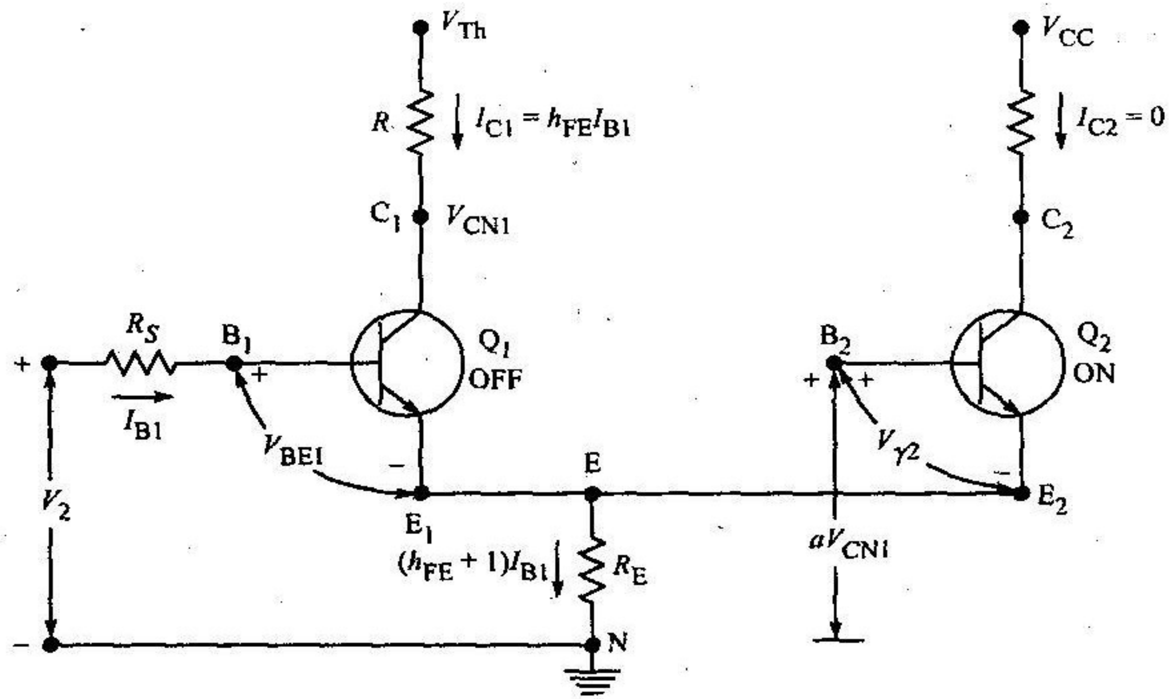
$$V_1 = V' - 0.1$$

Derivation of expression for LTP

Derivation of expression for LTP

The lower triggering point LTP is defined as the input voltage V2 at which the transistor Q2 resumes conduction.
*Vi can be calculated from the circuit shown in Figure 4.33 which is obtained by replacing Vcc, r*C1, *R1 and R2 of Figure 4.29 by Thevenin's equivalent voltage VTH and Thevenin's equivalent resistance R at the collector of Q 1, where*

$$V_{Th} = V_{CC}\frac{R_1 + R_2}{R_{C1} + R_1 + R_2} \quad \text{and} \quad R = R_{C1} \| (R_1 + R_2) = \frac{R_{C1}(R_1 + R_2)}{R_{C1} + R_1 + R_2}$$



The voltage ratio from the collector of Qi to the base of Q2 is

$$a = R_2/(R_1 + R_2).$$

The input signal to Q1 is decreasing, and when it reaches V2 then Q2 comes out of cut-off

$$aV_{CN1} - V_{\gamma 2} - (I_{B1} + I_{C1})R_E = 0$$

where

$$V_{CN1} = V_{Th} - I_{C1}R$$

$\therefore$

$$aV_{Th} - aI_{C1}R - V_{\gamma 2} - I_{C1}\left(1 + \frac{1}{h_{FE}}\right)R_E = 0$$

or

$$I_{C1} = \frac{aV_{Th} - V_{\gamma 2}}{aR + R_E\left(1 + \frac{1}{h_{FE}}\right)}$$

$\therefore$

$$aV_{Th} = \frac{R_2}{R_1 + R_2} \cdot V_{CC}\frac{R_1 + R_2}{R_1 + R_2 + R_{C1}} = V_{CC}\frac{R_2}{R_2 + R_{C1} + R_1} = V'$$

Let

$$R_E\left(1 + \frac{1}{h_{FE}}\right) = R'_E$$

$\therefore$

$$I_{C1} = \frac{V' - V_{\gamma 2}}{aR + R'_E}$$

Therefore from Figure 4.33,

$$V_2 = I_{B1}R_S + V_{BE1} + (I_{B1} + I_{C1})R_E$$

$$= V_{BE1} + I_{C1}\left(R_E\left(1+\frac{1}{h_{FE}}\right)+\frac{R_S}{h_{FE}}\right)$$

$$= V_{BE1} + I_{C1}\left(R'_E + \frac{R_S}{h_{FE}}\right)$$

$$= V_{BE1} + \frac{V' - V_{\gamma 2}}{aR + R'_E}\left(R'_E + \frac{R_S}{h_{FE}}\right)$$

Since h_{FE} is a large number, $R'_E \approx R_E$ and usually $\frac{R_S}{h_{FE}} << R_E$

$$\therefore \qquad V_2 = V_{BE1} + (V' - V_{\gamma 2})\frac{R'_E}{aR + R'_E}$$

MONOSTABLE MULTIVIBRATOR

A monostable multivibrator has got only one permanent stable state, the other state being quasi stable. Under quiescent conditions, the monostable multivibrator will be in its stable state only.

A triggering signal is required to induce a transition from the stable state to the quasi stable state

Once triggered properly the circuit may remain in its quasi stable state for a time which is very long compared with the time of transition between the states, and after that it will return to its original state.

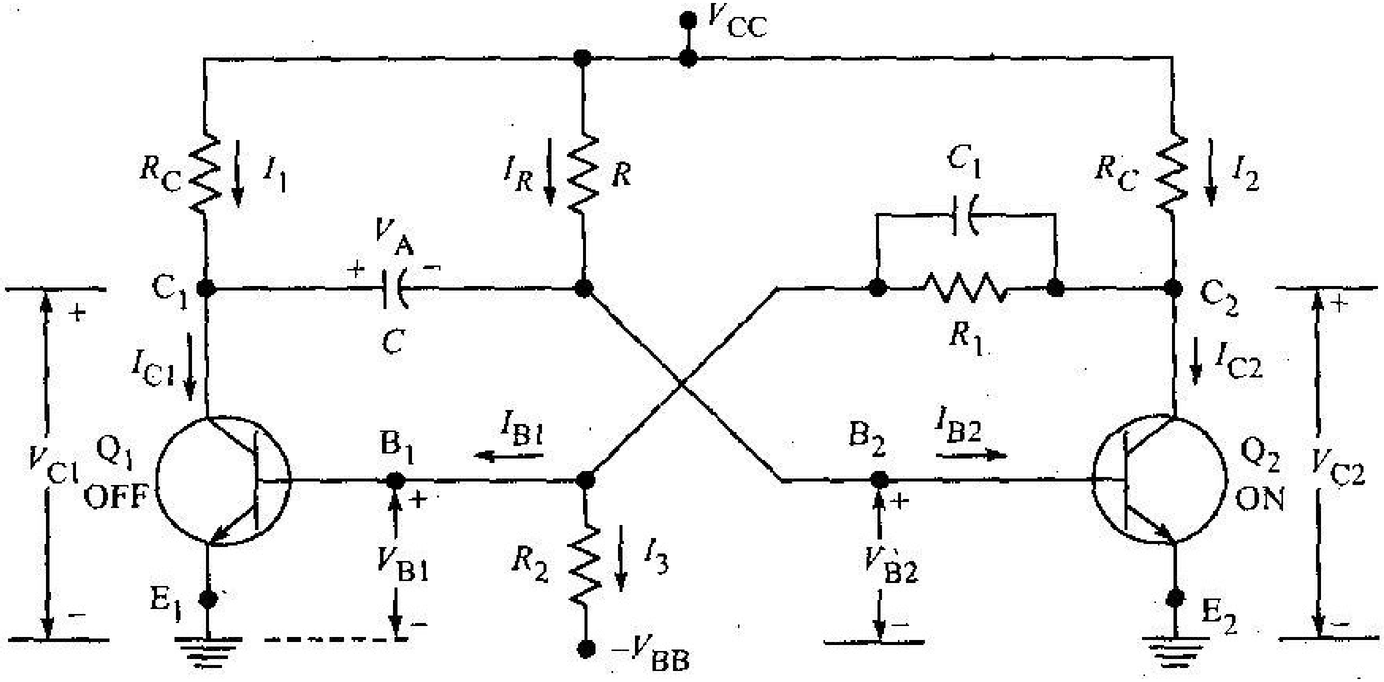
No external triggering signal is required to induce this reverse transition.

In a monostable multivibrator one coupling element is a resistor and another coupling element is a capacitor.

it generates a rectangular waveform which can be used to gate other circuits, it is also called a *gating circuit*

it generates a fast transition at a predetermined time *T after the input trigger, it is also referred to as a delay circuit.*

THE COLLECTOR COUPLED MONOSTABLE MULTIVIBRATOR



The transition from the stable state to the quasi-stable state takes place at *t = 0, and the reverse transition* from the quasi-stable state to the stable state takes place at *t = T.*

The time T for which the circuit is in its quasi-stable state is also referred to as the delay time and also as the gate width, pulse width, or pulse duration. The delay time may be varied by varying the time constant *τ= RC.*

Expression for the gate width T of a monostable multivibrator neglecting the reverse saturation current I_{CBO}

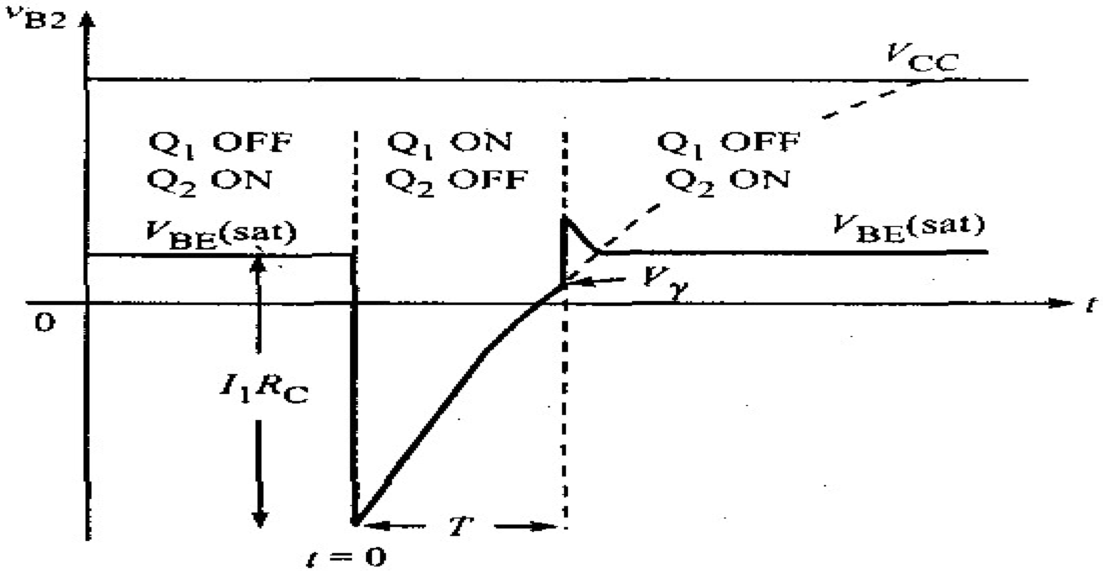
Figure 4.42(a) shows the waveform at the base of transistor Q2 of the monostable multivibrator shown in Figure 4.41.

For *t < 0, Q2 is ON and so vB2 = VBE(sat). At t = 0, a negative signal applied brings Q2 to OFF state and Q1 into saturation.*
A current I_1 flows through Rc of Q1 and hence vc1 drops abruptly by I_1R_c volts and so vB2 also drops by I_1R_c instantaneously. So at t = 0, vB2 = VBE(sat) – I_1R_C.

For t > 0, the capacitor charges with a time constant RC, and hence the base voltage of Q2 rises exponentially towards Vcc with the same time constant. At t = T, when this base voltage rises to the cut-in voltage level Vy of the transistor, Q2 goes to ON state, and Q1 to OFF state and the pulse ends

In the interval 0 < *t < T, the base voltage of Q2, i.e. vB2 is given by*

$$v_{B2} = V_{CC} - (V_{CC} - \{V_{BE}(\text{sat}) - I_1R_C\})e^{-t/\tau}$$



Voltage variation at the base of Q2 during the quasi-stable state

But $I_1R_C = V_{CC} - V_{CE}(\text{sat})$ (because at $t = 0^-$, $v_{C1} = V_{CC}$ and at $t = 0^+$, $v_{C1} = V_{CE}(\text{sat})$)

$\therefore$ $$v_{B2} = V_{CC} - [V_{CC} - \{V_{BE}(\text{sat}) - (V_{CC} - V_{CE}(\text{sat}))\}]e^{-t/\tau}$$

$$= V_{CC} - [2V_{CC} - \{V_{BE}(\text{sat}) + V_{CE}(\text{sat})\}]e^{-t/\tau}$$

At $$t = T,\ v_{B2} = V_\gamma$$

$\therefore$ $$V_\gamma = V_{CC} - [2V_{CC} - \{V_{CE}(\text{sat}) + V_{BE}(\text{sat})\}]e^{-T/\tau}$$

i.e. $$e^{T/\tau} = \frac{2V_{CC} - [V_{CE}(\text{sat}) + V_{BE}(\text{sat})]}{V_{CC} - V_\gamma}$$

$$\therefore \quad \frac{T}{\tau} = \frac{\ln\left[2\left(V_{CC} - \frac{V_{CE}(sat) + V_{BE}(sat)}{2}\right)\right]}{V_{CC} - V_\gamma}$$

i.e. $$T = \tau \ln 2 + \tau \ln \frac{V_{CC} - \frac{V_{CE}(sat) + V_{BE}(sat)}{2}}{V_{CC} - V_\gamma}$$

Normally for a transistor, at room temperature, the cut-in voltage is the average of the saturation junction voltages for either Ge or Si transistors

$$V_\gamma = \frac{V_{CE}(sat) + V_{BE}(sat)}{2}$$

Neglecting the second term in the expression for *T*

$$T = \tau \ln 2$$

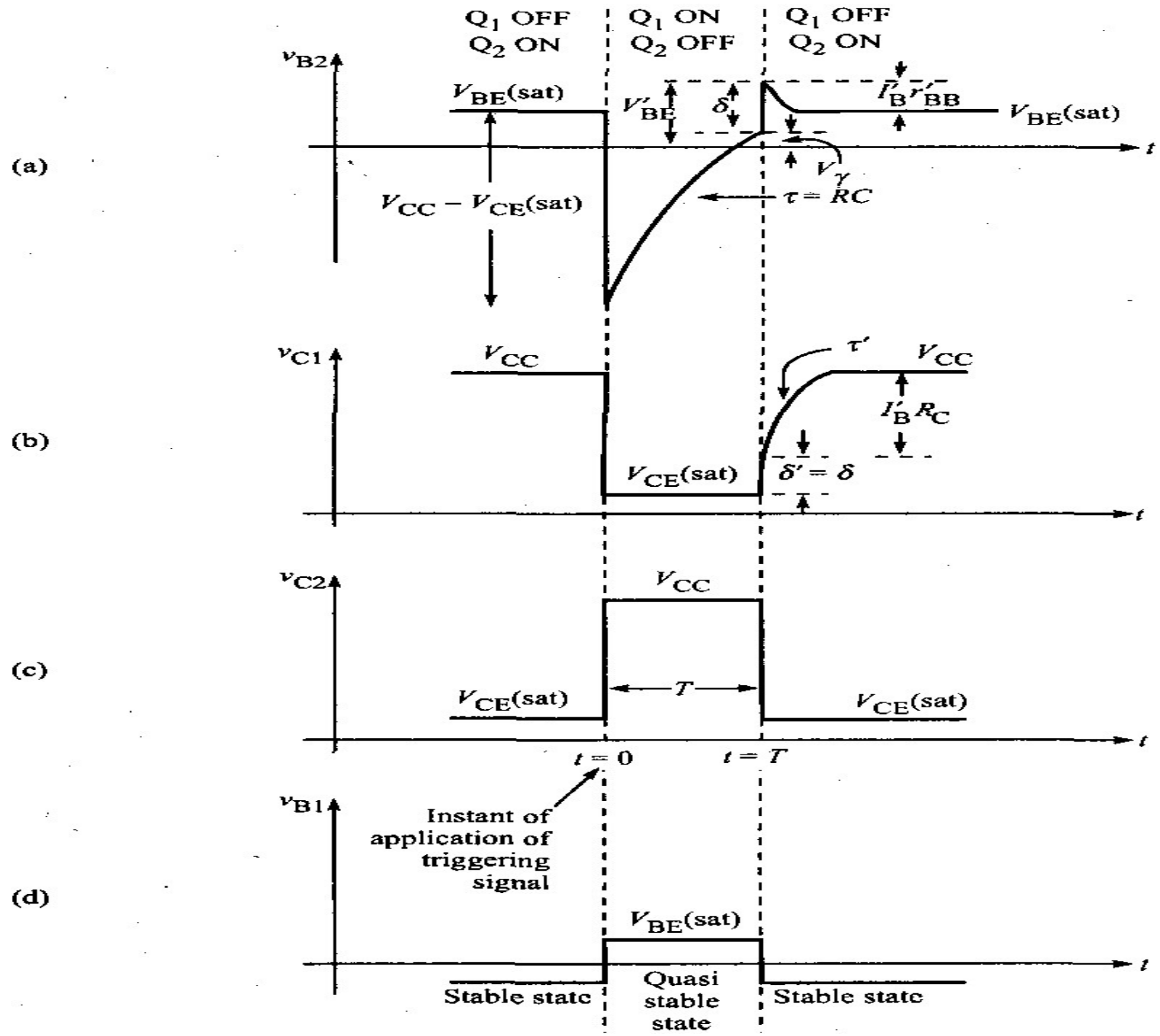
i.e. $$T = (R + R_o)C \ln 2 = 0.693(R + R_o)C$$

$\tau = R + R_o$

Where R_o is output impedance of conducting transistor

but for a transistor in saturation *Ro« R.*
Gate width, *T = 0.693RC*

Waveforms of the collector-coupled monostable multivibrator



Waveforms at the collectors and bases of the collector-coupled monostable multivibrator. (a) at the base of Q2, (b) at the collector of Q1, (c) at the collector of Q2, and (d) at the base Q1

The stable state:

For *t < 0, the monostable circuit is in its stable state with Q2 ON and Q1 OFF*

Since Q2 is ON, the base voltage of Q2 is vB2 = VBE2(sat) and the collector voltage of Q2 is vC2 = VCE2(sat).
Since Q1 is OFF, there is no current in *Rc of Q1 and its base voltage must be negative.*

Hence the voltage at the collector of Q1 is vC1 = VCC

The voltage at the base of Q1 using the superposition theorem is

$$v_{B1} = -V_{BB}\frac{R_1}{R_1 + R_2} + V_{CE2}(\text{sat})\frac{R_2}{R_1 + R_2}$$

The quasi-stable state:

A negative triggering signal applied at *t = 0 brings Q2 to OFF state and Q1 to ON state.*

A current I_1 flows in Rc of Q1. So, the collector voltage of Q1 drops suddenly by I_1R_C *volts.*

Since the voltage across the coupling capacitor C cannot change instantaneously, the voltage at the base of Q2 also drops by I_1R_C, where I_1R_C = Vcc -VcE2(sat)- Since Q1 is ON,

$$v_{B1} = V_{BE1}(\text{sat}) \quad \text{and} \quad v_{C1} = V_{CE1}(\text{sat})$$

$$\text{Also,} \quad v_{B2} = V_{BE2}(\text{sat}) - I_1R_C \quad \text{and} \quad v_{C2} = V_{CC}\frac{R_1}{R_1 + R_C} + V_{BE1}(\text{sat})\frac{R_C}{R_1 + R_C}$$

In the interval 0 < *t < T, the voltages Vc1, VB1 and Vc2 remain constant at their values at t = 0,*

but the voltage at the base of Q2, i.e. vB2 rises exponentially towards Vcc with a time constant, τ=RC, until at t = T, vB2 reaches the cut-in voltage V_γ of the transistor

Waveforms for $t > T$:

At $t = T^+$, reverse transition -takes place. Q2 conducts and Q1 is cut-off.
The collector voltage of Q2 and the base voltage of Q1 return to their voltage levels for t< *0.*

The voltage vc1 now rises abruptly since Q1 is OFF. This increase in voltage is transmitted to the base of Q2 and drives Q2 heavily into saturation.

Hence an overshoot develops in vB2 at *t =* T^+ *, which decays as the capacitor recharges because of the base current.*

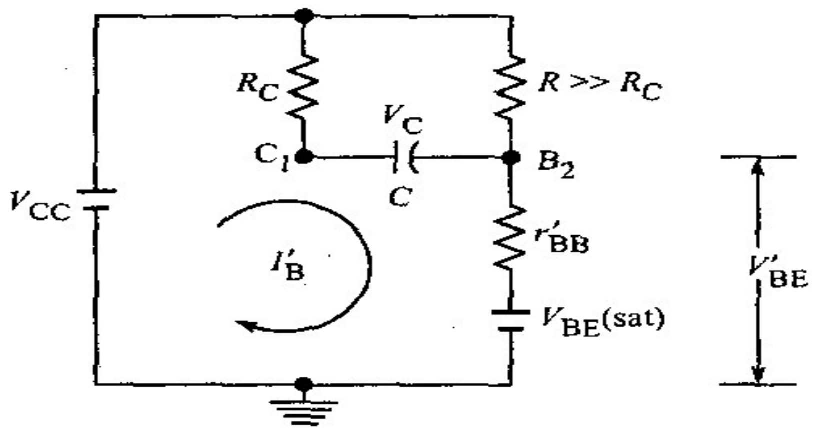
The magnitude of the base current may be calculated as follows.

Replace the input circuit of Q2 by the base spreading resistance rBB in series with the voltage VBE(sat) as shown in Figure.

Let I'B be the base current at *t =* T^+*. The current in R may be neglected compared to I'B.*

From the below Figure

$$V'_{BE} = I'_B r'_{BB} + V_{BE}(\text{sat}) \quad \text{and} \quad V_C = V_{CC} - I'_B R_C - V'_{BE}$$



Equivalent circuit for calculating the overshoot at base B2 of Q2

The jumps in voltages at B2 and C1 are, respectively, given by

$$\delta \approx V'_{BE} - V_\gamma \approx I'_B r'_{BB} + V_{BE}(\text{sat}) - V_\gamma \quad \text{and} \quad \delta' = V_{CC} - V_{CE}(\text{sat}) - I'_B R_C$$

Since C1 and B2 are connected by a capacitor C and since the voltage across the capacitors cannot change instantaneously, these two discontinuous voltage changes δ and δ' must be equal. Equating them

$$I'_B r'_{BB} + V_{BE}(\text{sat}) - V_\gamma = V_{CC} - V_{CE}(\text{sat}) - I'_B R_C$$

$$I'_B \approx \frac{V_{CC} - V_{BE}(\text{sat}) - V_{CE}(\text{sat}) + V_\gamma}{R_C + r'_{BB}}$$

vB2 and vcl decay to their steady-state values with a time constant $\tau' \approx (R_C + r'_{BB})C$

Monostable multivibrator as a voltage-to-time converter (as a pulse width modulator)

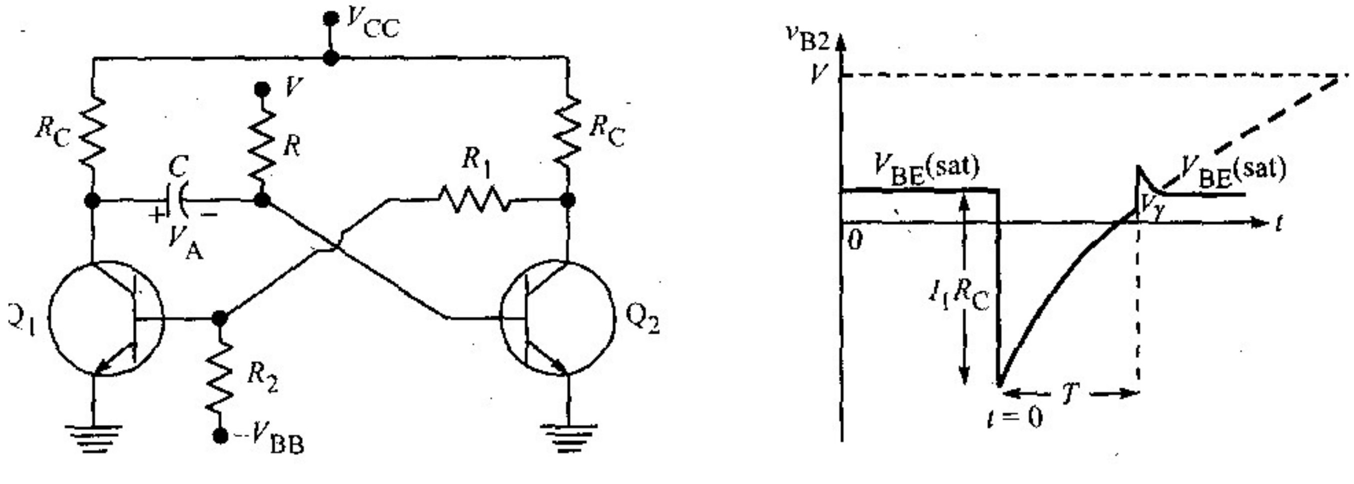


Figure shows the circuit diagram of a monostable multivibrator as a voltage- to-time converter.

By varying the auxiliary supply voltage *V, the pulse width can be changed. It can* be seen that the resistor *R is connected to the auxiliary voltage source V instead of to Vcc.*

The waveform of the voltage vB2 at the base of Q2 is shown in Figure 4.45(b).

In the interval $0 < t < T$, v_{B2} is given by

$$v_{B2} = v_f - (v_f - v_{in})\, e^{-t/\tau}$$

$$\therefore \quad v_{B2} = V - [V - (V_{BE}(\text{sat}) - I_1 R_C)]e^{-t/\tau}$$

But $I_1 R_C = V_{CC} - V_{CE}$ (sat)

$$\therefore \quad v_{B2} = V - [V - \{V_{BE}(\text{sat}) - (V_{CC} - V_{CE}(\text{sat}))\}]\, e^{-t/\tau}$$
$$= V - [V + V_{CC} - (V_{BE}(\text{sat}) + V_{CE}(\text{sat}))]e^{-t/\tau}$$

At $t = T$, $v_{B2} = V_\gamma$

$$\therefore \quad V_\gamma = V - [V + V_{CC} - (V_{BE}(\text{sat}) + V_{CE}(\text{sat}))]e^{-T/\tau}$$

$$\therefore \quad e^{T/\tau} = \frac{V + V_{CC} - (V_{BE}(\text{sat}) + V_{CE}(\text{sat}))}{V - V_\gamma}$$

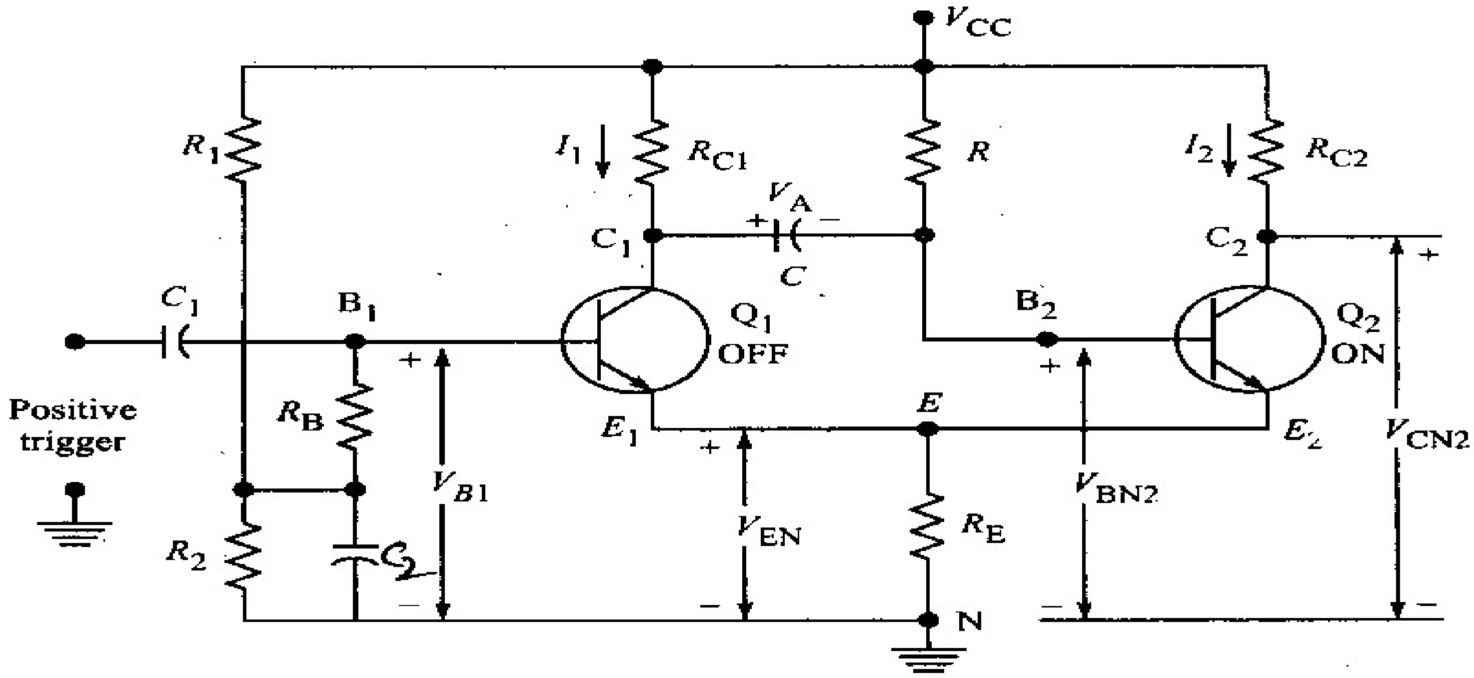
Neglecting the junction voltages and the cut in voltage

$$T = \tau \log \frac{V + V_{CC}}{V}$$

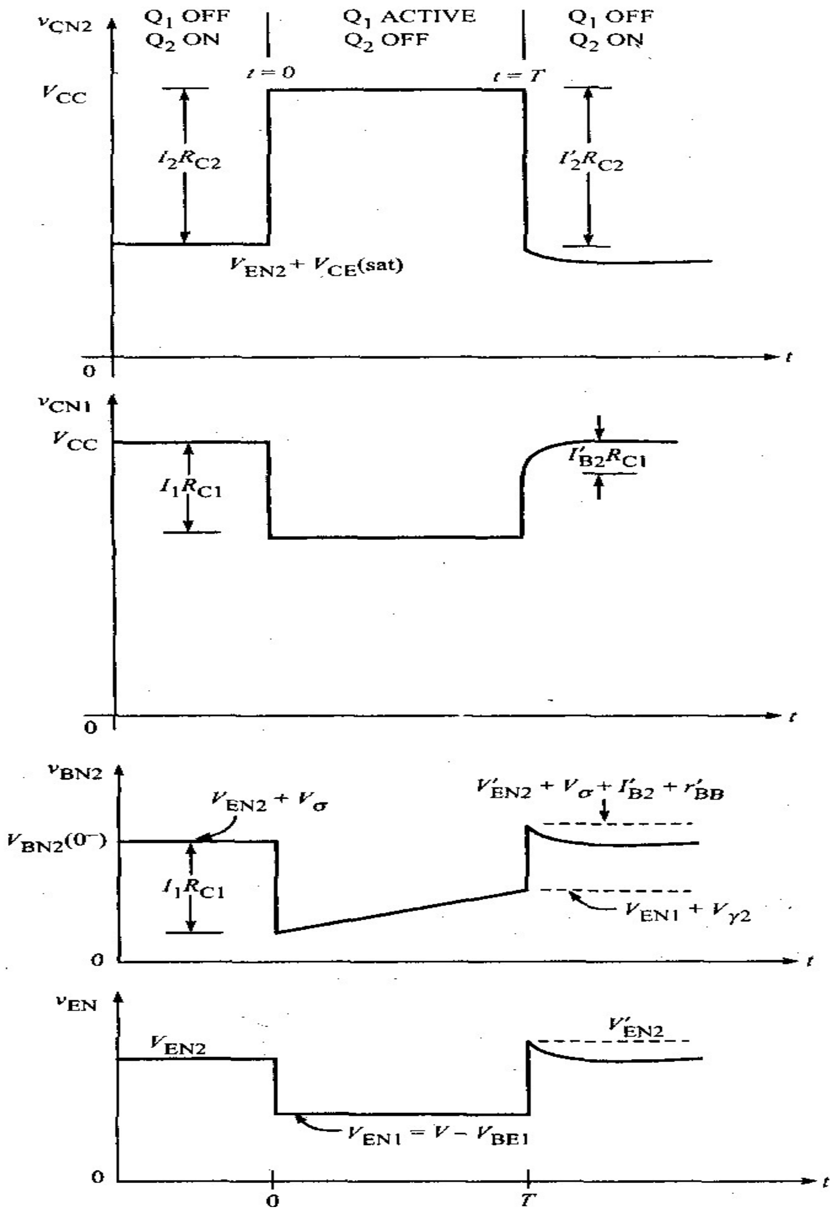
$$= \tau \log\left(1 + \frac{V_{CC}}{V}\right)$$

Thus the pulse width is a function of auxiliary voltage V. For this reason the monostable multivibrator shown in Figure is termed a *voltage-to-time converter. It is also called a pulse width modulator.*

THE EMITTER-COUPLED MONOSTABLE MULTIVIBRATOR



Waveforms of emitter-coupled monostable multivibrator



ASTABLE MULTIVIBRATOR

As the name indicates an astable multivibrator is a multivibrator with no permanent stable state. Both of its states are quasi stable only.

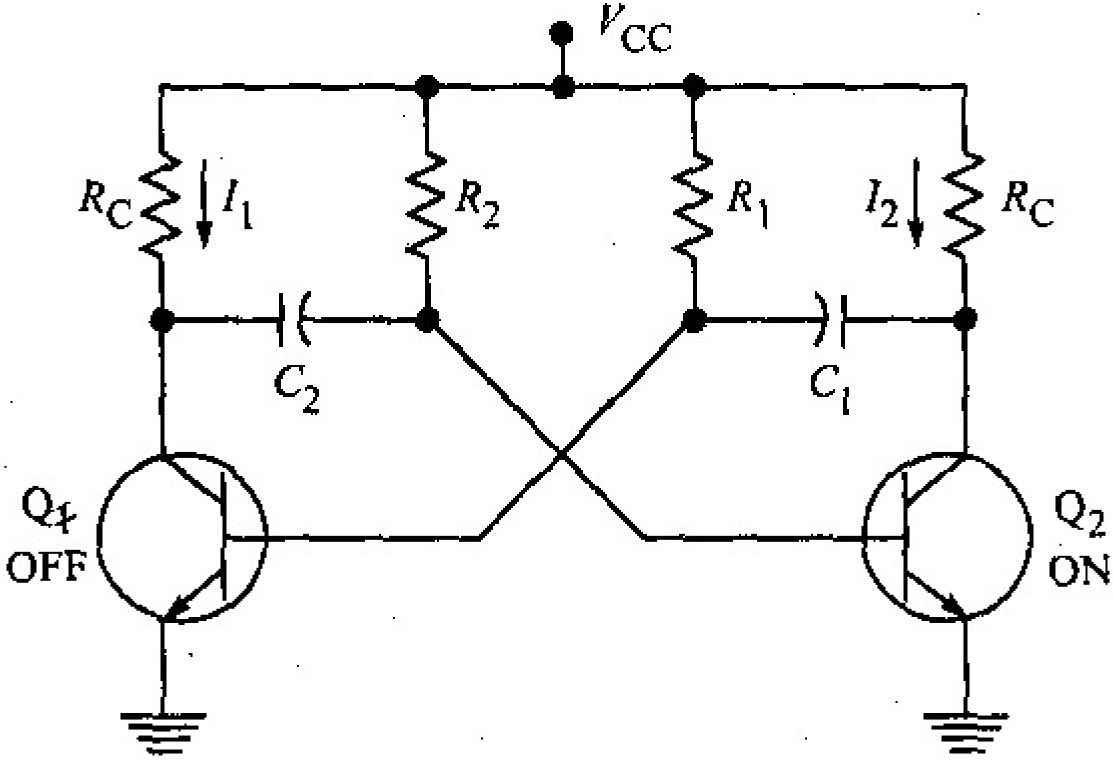
It cannot remain in any one of its states indefinitely and keeps on oscillating between its two quasi stable states the moment it is connected to the supply.

It remains in each of its two quasi stable states for only a short designed interval of time and then goes to the other quasi stable state.

No triggering signal is required. Both the coupling elements are capacitors (ac coupling) and hence both the states are quasi stable.

It is a free running multivibrator. It generates square waves.
It is used as a master oscillator.

THE COLLECTOR-COUPLED ASTABLE MULTIVIBRATOR



Expression for the frequency of oscillation of an astable multivibrator

Consider the waveform at the base of Q_1 shown in Figure 4.54(d). At $t = 0$,

$$v_{B1} = V_{BE}(sat) - I_2R_C$$

But

$$I_2R_C = V_{CC} - V_{CE}(sat)$$

$\therefore$

$$\text{At } t = 0,\ v_{B1} = V_{BE}(sat) - V_{CC} + V_{CE}(sat)$$

For $0 < t < T_1$, v_{B1} rises exponentially towards V_{CC} given by the equation,

$$v_o = v_f - (v_f - v_i)e^{-t/\tau}$$

$\therefore$

$$v_{B1} = V_{CC} - [V_{CC} - (V_{BE}(sat) - V_{CC} + V_{CE}(sat))]e^{-t/\tau_1}, \text{ where } \tau_1 = R_1C_1$$

At $t = T_1$, when v_{B1} rises to V_γ, Q_1 conducts

$\therefore$

$$V_\gamma = V_{CC} - [2V_{CC} - (V_{BE}(sat) + V_{CE}(sat))]e^{-T_1/R_1C_1}$$

or

$$e^{T_1/R_1C_1} = \frac{2\left[V_{CC} - \dfrac{V_{BE}(sat) + V_{CE}(sat)}{2}\right]}{V_{CC} - V_\gamma}$$

$$T_1 = R_1C_1 \ln \frac{2\left[V_{CC} - \dfrac{V_{CE}(\text{sat}) + V_{BE}(\text{sat})}{2}\right]}{V_{CC} - V_\gamma}$$

$$T_1 = R_1C_1 \ln 2 + R_1C_1 \ln \frac{\left[V_{CC} - \dfrac{V_{CE}(\text{sat}) + V_{BE}(\text{sat})}{2}\right]}{V_{CC} - V_\gamma}$$

At room temperature for a transistor,

$$V_\gamma \simeq \frac{V_{CE}(\text{sat}) + V_{BE}(\text{sat})}{2}$$

$$\therefore \quad T_1 = R_1C_1 \ln 2 \simeq 0.693R_1C_1$$

On similar lines considering the waveform of Figure , we can show that the time *T2 for which Q2 is OFF and Q1 is ON is given by*

$$T_2 = R_2C_2 \ln 2 \simeq 0.693R_2C_2$$

$$T = T_1 + T_2 = 0.693(R_1C_1 + R_2C_2)$$

The frequency of oscillation $f = \frac{1}{T} = \frac{1}{0.693(R_1C_1 + R_2C_2)}$

If *R1 = R2 = R, and C1 = C2 = C, then TI = T2 = T/2*

$$T = 2 \times 0.693RC = 1.386RC \quad \text{and} \quad f = \frac{1}{1.386RC}$$

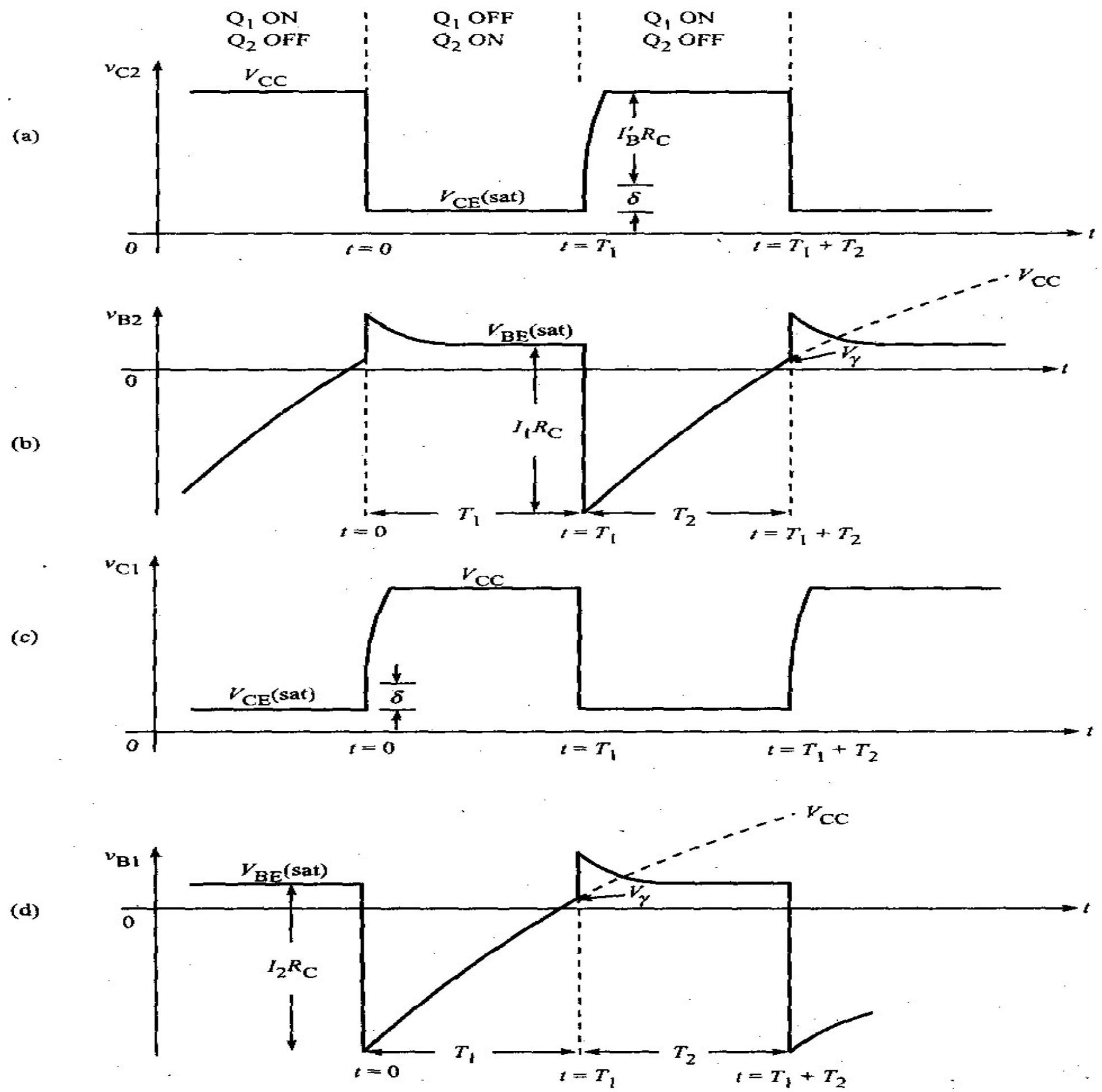
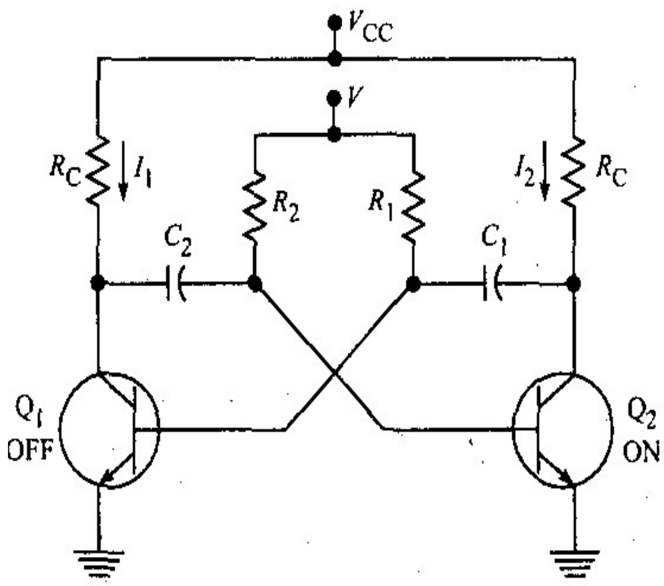
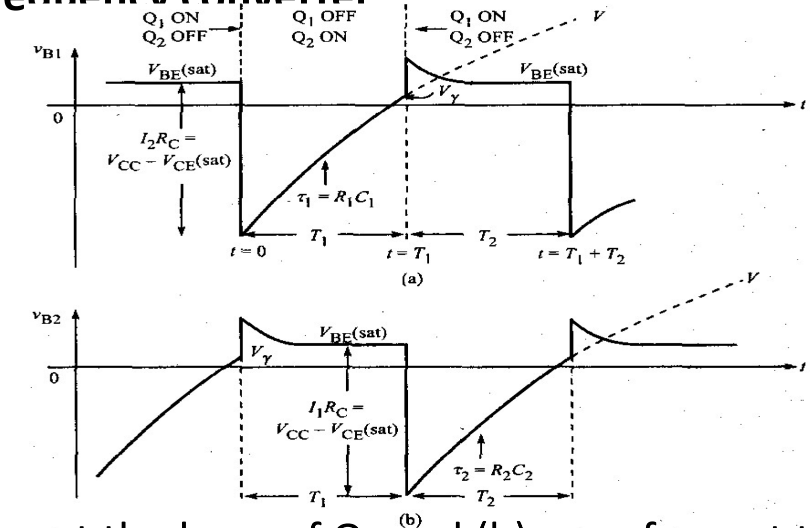


Figure 4.54 Waveforms at the bases and collectors of a collector-coupled astable multivibrator.

The astable multivibrator as a voltage-to-frequency converter





(a) Waveform at the base of Q, and (b) waveform at the base of Q2 for the circuit of Figure

Figure shows the circuit diagram of an astable multivibrator used as a voltage-to-frequency converter.

The frequency can be varied by varying the magnitude of the auxiliary voltage source *V.*

Now the supply voltage is VCC only, but the voltage level to which the coupling capacitors C1 and C2 try to charge is not Vcc but V.

For $0 < t < T1$, *Q1 is OFF and Q2 is ON. From the base waveform shown in Figure the voltage at the base of Q1 is given by*

$$v_{B1} = V - [V - (V_{BE}(\text{sat}) - I_2R_C)]e^{-t/\tau_1}$$
$$= V - [V - \{V_{BE}(\text{sat}) - (V_{CC} - V_{CE}(\text{sat})\}]e^{-T_1/\tau_1}$$

At $t = T_1$, $v_{B1} = V_\gamma$, and the transistor Q_1 conducts

$$\therefore \quad V_\gamma = V - [V + V_{CC} - (V_{BE}(\text{sat}) + V_{CE}(\text{sat}))]e^{-T_1/\tau_1}$$

or

$$e^{T_1/\tau_1} = \frac{(V + V_{CC}) - (V_{BE}(\text{sat}) + V_{CE}(\text{sat}))}{V - V_\gamma}$$

Neglecting the junction voltages and the cut-in voltage of the transistor

$$e^{T_1/\tau_1} = \frac{V + V_{CC}}{V} = 1 + \frac{V_{CC}}{V}$$

$\therefore$ $$T_1 = \tau_1 \ln\left(1 + \frac{V_{CC}}{V}\right) = R_1C_1 \ln\left(1 + \frac{V_{CC}}{V}\right)$$

Similarly, considering the waveform at the base of Q_2 shown in Figure 4.56(b)

$$T_2 = R_2C_2 \ln\left(1 + \frac{V_{CC}}{V}\right)$$

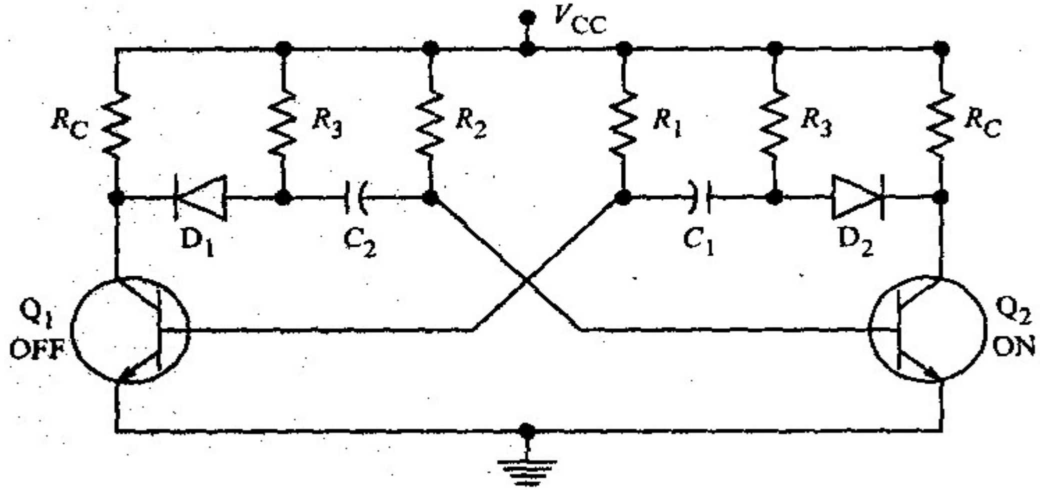
$\therefore$ The period, $$T = T_1 + T_2 = (R_1C_1 + R_2C_2) \ln\left(1 + \frac{V_{CC}}{V}\right)$$

If $R_1 = R_2 = R$, $C_1 = C_2 = C$, then $T_1 = T_2 = T/2$

$\therefore$ $$T = 2RC \ln\left(1 + \frac{V_{CC}}{V}\right)$$

or $$f = \frac{1}{2RC \ln\left(1 + \frac{V_{CC}}{V}\right)}$$

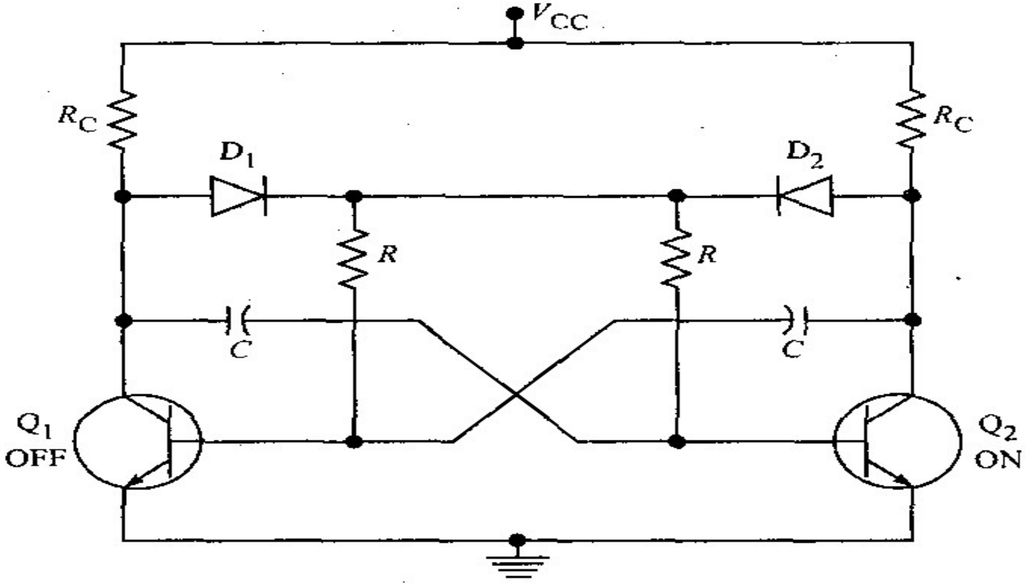
The astable multivibrator with vertical edges



The collector-coupled astable multivibrator produces the output waveforms at the collectors of Q1 and Q2 with rounded edges .
An astable multivibrator which can generate collector waveforms with vertical edges can be obtained by the addition of two diodes and two resistors as shown in Figure above.

If Q2 is driven OFF, its collector voltage rises immediately to Vcc. so that D2 is reverse biased and Q1 goes into saturation, The saturation base current of Q1 passes through $C_1$and R_3 *rather than through* R_c. *Since* I_B *no longer passes through Rc, the collector waveform now has vertical edges as desired.*

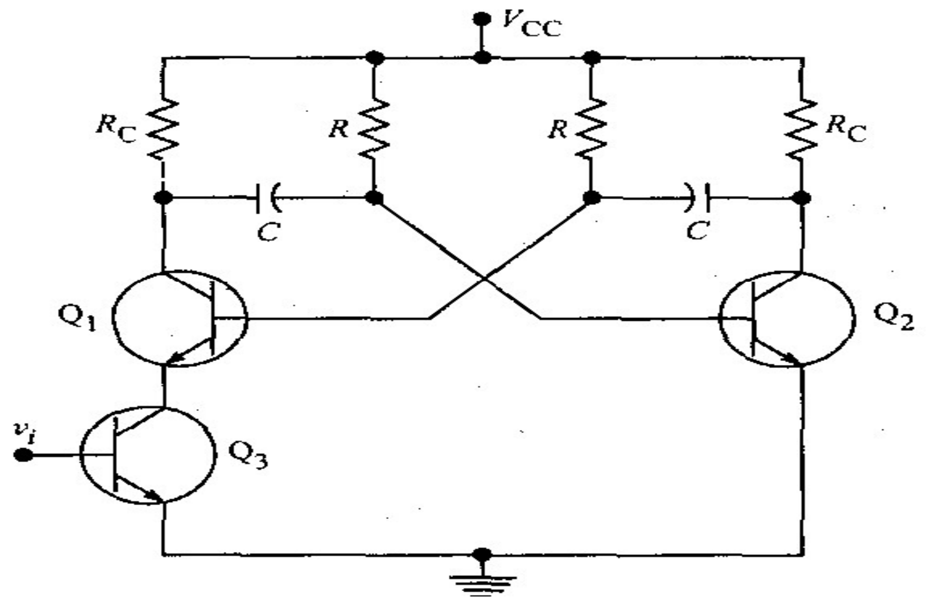
The astable multivibrator which does not block



For the astable multivibrator , if the supply voltage is increased slowly from zero to its full value Vcc, both the transistors may go into saturation simultaneously and remain in that state. This *blocked condition does not occur if the voltage is applied suddenly. A circuit which cannot block is shown in Figure above.*

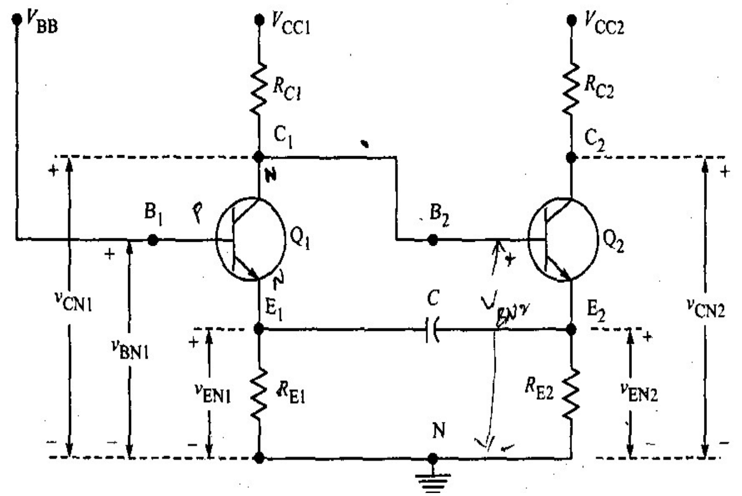
The gated astable multivibrator

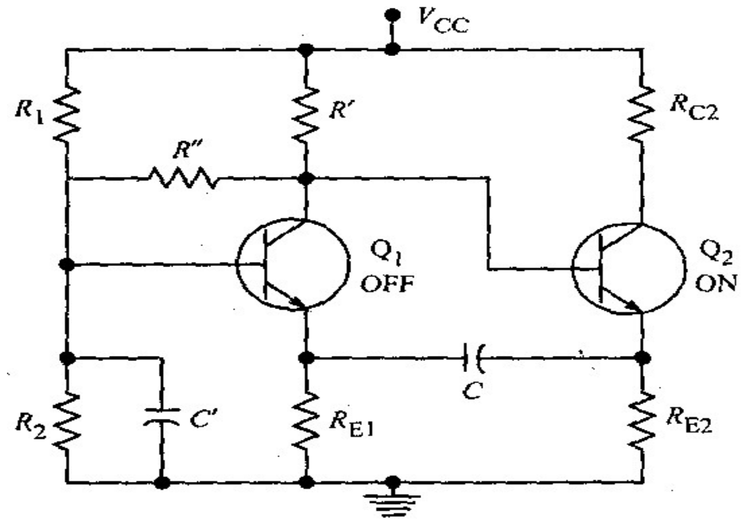
Figure 4.59 shows the circuit diagram of a gated astable multivibrator. This is obtained by adding a transistor Q3 in series with the emitter of Q| or Q2 of the collector-coupled astable multivibrator. This gated astable multivibrator can start or stop oscillating at definite times.



The input v_i to Q3 can assume one of two values. One level is chosen such that Q3 is OFF. With Q3 OFF, Q1 will be OFF, and Q2 will be ON and the circuit is quiescent, i.e. it does not oscillate. The second binary level is chosen such that Q_3 is driven into saturation. Hence, at any instant (say *t - 0) that this voltage is applied, Q, goes ON and Q2 is driven OFF.*

THE EMITTER-COUPLED ASTABLE MULTIVIE





The emitter-coupled multivibrator

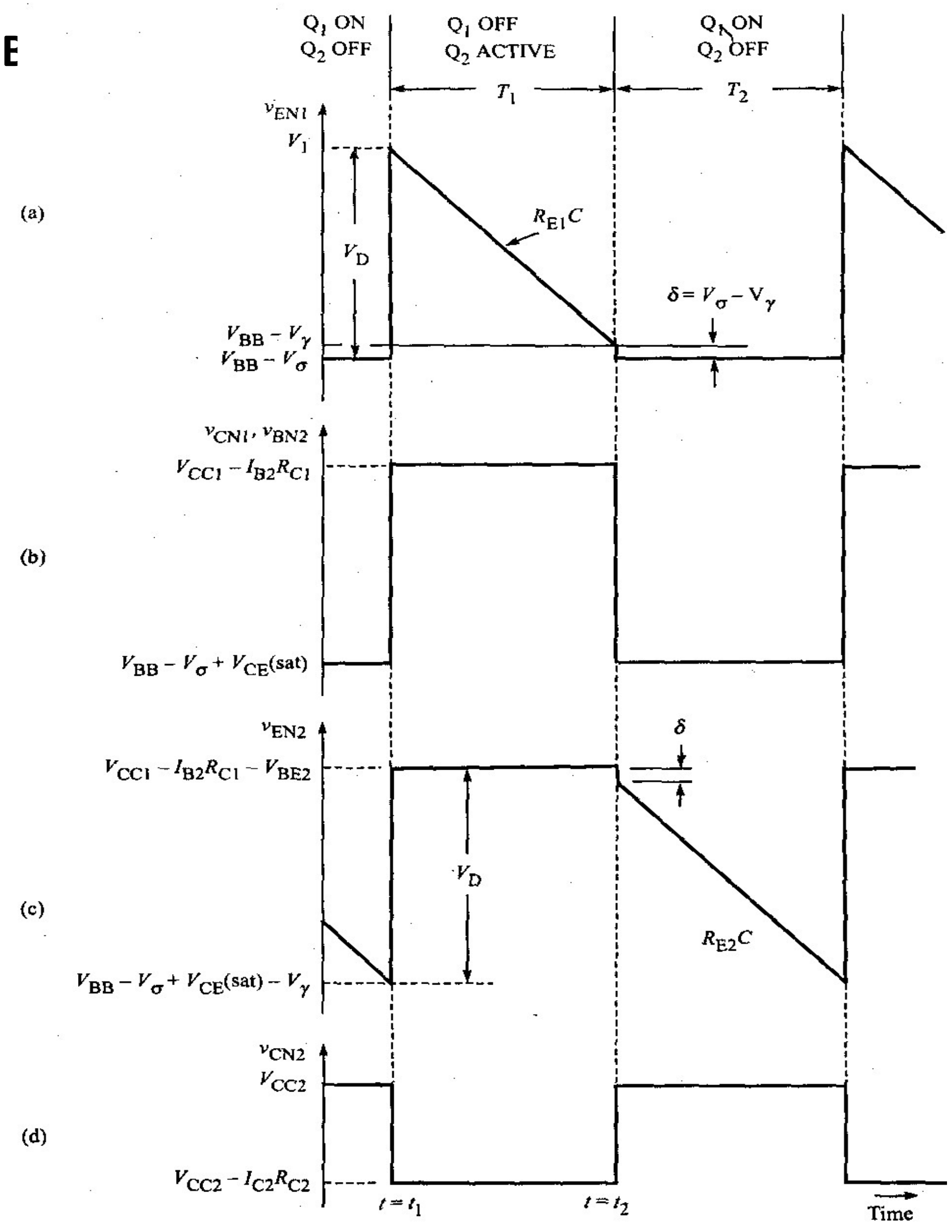


Figure 4.64 Waveforms of the emitter-coupled astable multivibrator.

Advantages

1. It is inherently self-starting.
2. The collector of Q2 where the output is taken may be loaded heavily even capacitively.
3. The output is free of recovery transients.
4. Because it has an isolated input at the base of Q1, synchronization is convenient.
5. Frequency adjustment is convenient because only one capacitor is use

Disadvantages

1. This circuit is more difficult to adjust for proper operating conditions.
2. This circuit cannot be operated with *T\ and T2 widely different.*
3. This circuit uses more components than does the collector-coupled circuit.

TIME BASE GENERATORS

A time-base generator is an electronic circuit which generates an output voltage or current waveform, a portion of which varies linearly with time.

Ideally the output waveform should be a ramp

Time-base generators may be :

voltage time-base generators

or current time-base generators.

A **voltage time-base generator** is one that provides an output voltage waveform, a portion of which exhibits a linear variation with respect to time.

A **current time-base generator** is one that provides an output current waveform, a portion of which exhibits a linear variation with respect to time.

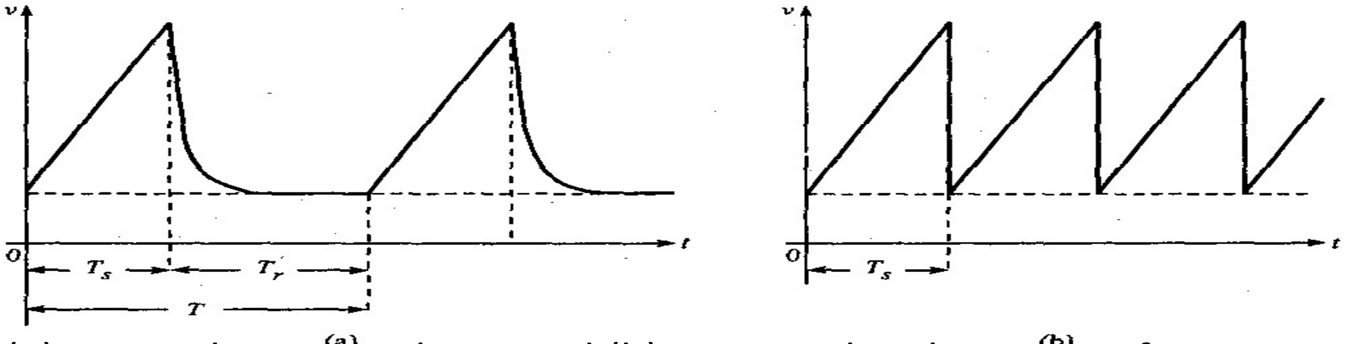
Applications of time-base generators:

such as in CROs, television and radar displays, in precise time measurements, and in time modulation.

The most important application of a time-base generator is in CROs. To display the variation with respect to time of an arbitrary waveform on the screen of an oscilloscope it is required to apply to one set of deflecting plates a voltage which varies linearly with time.

Since this waveform is used to sweep the electron beam horizontally across the screen it is called the *sweep voltage and the time-base generators are called the sweep circuits.*

GENERAL FEATURES OF A TIME-BASE SIGNAL



(a) General sweep voltage and (b) saw-tooth voltage waveforms

Figure 5.1(a) shows the typical waveform of a time-base voltage. As seen the voltage starting from some initial value increases linearly with time to a maximum value after which it returns again to its initial value.

The time during which the output increases is called the *sweep time and the time taken by the signal to return to its initial value is called the restoration time, the return time, or the flyback time.*

However, in some cases a restoration time which is very small compared with the sweep time is required.

If the restoration time is almost zero and then the waveform is known as Saw-tooth voltage shown in in figure. b

The waveforms of the type shown in Figures 5.1 (a) and (b) are generally called sweep waveforms even when they are used in applications not involving the deflection of an electron beam

The deviation from linearity is expressed in three most important ways:

1 . The slope or sweep speed error, e_s

2. The displacement error, e_d

3. The transmission error, e_t

The slope or sweep-speed error, *es*

An important requirement of a sweep is that it must increase linearly with time, i.e. the rate of change of sweep voltage with time be constant.

This deviation from linearity is defined as

$$\text{Slope or sweep-speed error, } e_s = \frac{\text{difference in slope at beginning and end of sweep}}{\text{initial value of slope}}$$

$$= \frac{\left.\dfrac{dv_0}{dt}\right|_{t=0} - \left.\dfrac{dv_0}{dt}\right|_{t=T_s}}{\left.\dfrac{dv_0}{dt}\right|_{t=0}}$$

The displacement error, e_d

Another important criterion of linearity is the maximum difference between the actual sweep voltage and the linear sweep which passes through the beginning and end points of the actual sweep.

The displacement error e_d *is defined as*

$$e_d = \frac{\text{maximum difference between the actual sweep voltage and the linear sweep which passes through the beginning and end points of the actual sweep}}{\text{amplitude of the sweep at the end of the sweep time}}$$

$$= \frac{(v_s - v'_s)_{max}}{V_s}$$

v_s is the actual sweep and v's is the linear sweep

The transmission error, e_t

When a ramp signal is transmitted through a high-pass circuit, the output falls away from the input as shown in Figure 5.2(b). This deviation is expressed as transmission error *et, defined as the difference between the input and the output divided by the input at the end of the sweep*

$$e_t = \frac{V'_s - V_s}{V'_s}$$

where as shown in Figure 5.2(b), *V's is the input and Vs is the output at the end of the sweep, i.e.* at *t = TS*

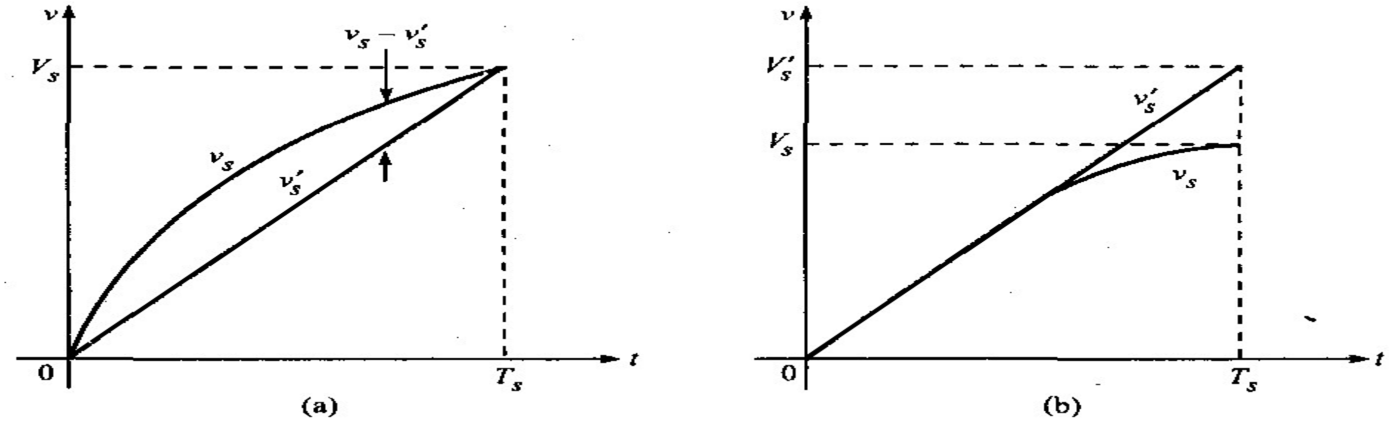


Figure 5.2 (a) Sweep for displacement error and (b) sweep for transmission error.

If the deviation from linearity is small so that the sweep voltage may be approximated by the sum of linear and quadratic terms in *t, then the above three errors are related as*

$$e_d = \frac{e_s}{8} = \frac{e_t}{4}$$

$$e_s = 2e_t = 8e_d$$

which implies that the sweep speed error is the more dominant one and the displacement error is the least severe one.

METHODS OF GENERATING A TIME-BASE WAVEFORM

In time-base circuits, sweep linearity is achieved by one of the following methods

1. ***Exponential charging****. In this method a capacitor is charged from a supply voltage through a resistor to a voltage which is small compared with the supply voltage.*

2. ***Constant current charging****. In this method a capacitor is charged linearly from a constant current source. Since the charging current is constant the voltage across the capacitor increases linearly.*

3. ***The Miller circuit****. In this method an operational integrator is used to convert an input step voltage into a ramp waveform.*

4. ***The Phantastron circuit****. In this method a pulse input is converted into a ramp. This is a version of the Miller circuit.*

5. ***The bootstrap circuit****. In this method a capacitor is charged linearly by a constant current which is obtained by maintaining a constant voltage across a fixed resistor in series with the capacitor.*

*6****. Compensating networks****. In this method a compensating circuit is introduced to improve the linearity of the basic Miller and bootstrap time-base generators.*

7. ***An inductor circuit****. In this method an RLC series circuit is used. Since an inductor does not allow the current passing through it to change instantaneously, the current through the capacitor more or less remains constant and hence a more linear sweep is obtained.*

EXPONENTIAL SWEEP CIRCUIT

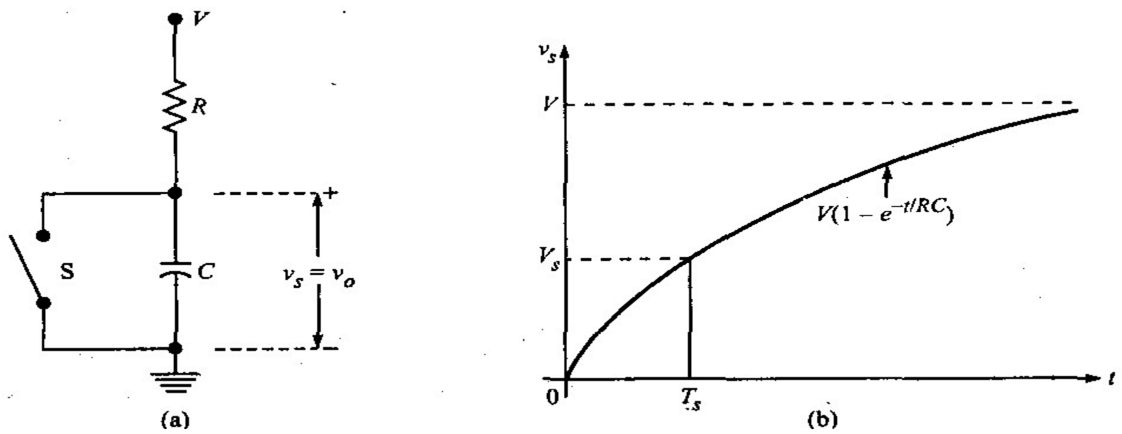


Figure 5.3(a) shows an exponential sweep circuit. The switch S is normally closed and is open at *t = 0. So for t > 0, the capacitor charges towards the supply voltage V with a time constant RC.*

The voltage across the capacitor at any instant of time is given by

$$v_o(t) = V(1 - e^{-t/RC})$$

Slope or sweep **speed error, *es***

We know that for an exponential sweep circuit of Figure 5.3(a),

$$v_o(t) = V(1 - e^{-t/RC})$$

Rate of change of output or slope is

$$\frac{dv_o}{dt} = 0 - V(e^{-t/RC})\left(\frac{-1}{RC}\right) = \frac{Ve^{-t/RC}}{RC}$$

$$\therefore \quad \text{Slope error, } e_s = \frac{\left.\frac{dv_o}{dt}\right|_{t=0} - \left.\frac{dv_o}{dt}\right|_{t=T_s}}{\left.\frac{dv_o}{dt}\right|_{t=0}} = \frac{\frac{V}{RC} - \frac{Ve^{-T_s/RC}}{RC}}{\frac{V}{RC}}$$

$$= 1 - e^{-T_s/RC}$$

$$= 1 - \left(1 - \frac{T_s}{RC} + \left(\frac{-T_s}{RC}\right)^2 \frac{1}{2} + \ldots\right)$$

For small *Ts, neglecting the second and higher order terms*

$$e_s = \frac{T_s}{RC}$$

Also, $$v_o = V(1 - e^{-t/RC})$$

At $$t = T_s, \qquad v_o = V_s$$

$\therefore$ $$V_s = V(1 - e^{-T_s/RC}) = V\left[1 - \left(1 - \frac{T_s}{RC} + \left(-\frac{T_s}{RC}\right)^2 \frac{1}{2!} + \ldots\right)\right]$$

Neglecting the second and higher order terms

$$V_s = V\frac{T_s}{RC} \quad \text{or} \quad \frac{V_s}{V} = \frac{T_s}{RC}$$

Hence $$\frac{V_s}{V} = \frac{T_s}{RC} = e_s$$

So the smaller the sweep amplitude compared to the sweep voltage, the smaller will be the slope error.

The transmission error, e_t :

From Figure 5.2(b),

$$v_s = V(1 - e^{-t/RC})$$

At $t = T_s$,

$$v_s = V_s = V(1 - e^{-T_s/RC})$$

$$= V\left[1 - \left(1 - \frac{T_s}{RC} + \left(-\frac{T_s}{RC}\right)^2 \frac{1}{2!} + \ldots\right)\right]$$

$$V_s = V\left(\frac{T_s}{RC} - \frac{1}{2}\left(\frac{T_s}{RC}\right)^2\right)$$

The initial slope, $\left.\frac{dv_o}{dt}\right|_{t=0} = \frac{V}{RC}$

If the initial slope is maintained at $t = T_s$, $v_s = V_s' = T_s \times \frac{V}{RC}$

$$e_t = \frac{V_s' - V_s}{V_s'} = \frac{\frac{VT_s}{RC} - \left(\frac{VT_s}{RC} - \frac{V}{2}\left(\frac{T_s}{RC}\right)^2\right)}{\frac{VT_s}{RC}} = \frac{T_s}{2\,RC} = \frac{e_s}{2}$$

The displacement error, e_d

From Figure 5.2(a), we can see that the maximum displacement between the actual sweep and the linear sweep which passes through the beginning and end points of the actual sweep occurs at *t =T S / 2*

$$\text{At} \quad t = \frac{T_s}{2}, \quad v_s' = \frac{V_s}{2}$$

The actual sweep *vs is given by*

$$v_s = V(1 - e^{-t/RC})$$

At $t = \dfrac{T_s}{2}$

$$v_s = V(1 - e^{-T_s/2RC})$$

$$= V\left[1 - \left\{1 - \frac{T_s}{2RC} + \left(-\frac{T_s}{2RC}\right)^2 \frac{1}{2!} + \ldots\right\}\right]$$

$$= V\left[\frac{T_s}{2RC} - \left(\frac{T_s}{RC}\right)^2 \frac{1}{8}\right]$$

At $t = T_s$, $\quad v_o = V_s$

$\therefore \quad V_s = V(1 - e^{-T_s/RC})$

$$= V\left[1 - \left\{1 - \frac{T_s}{RC} + \left(-\frac{T_s}{RC}\right)^2 \frac{1}{2!} + \ldots\right\}\right]$$

$$= V\left[\frac{T_s}{RC} - \frac{1}{2}\left(\frac{T_s}{RC}\right)^2\right]$$

The displacement error *ed is given by*

$$e_d = \frac{(v_s - v_s')_{\max}}{V_s} = \frac{V\left[\frac{T_s}{2RC} - \frac{1}{8}\frac{T_s^2}{(RC)^2}\right] - \frac{V}{2}\left[\frac{T_s}{RC} - \frac{T_s^2}{2(RC)^2}\right]}{V\left[\frac{T_s}{RC} - \left(\frac{T_s}{RC}\right)^2 \frac{1}{2}\right]}$$

$$= \frac{\frac{V}{2}\left[-\frac{T_s^2}{4(RC)^2} + \frac{T_s^2}{2(RC)^2}\right]}{V\left[\frac{T_s}{RC}\right]}$$

$$= \frac{1}{2}\left[\frac{\frac{1}{4}\left(\frac{T_s}{RC}\right)^2}{\frac{T_s}{RC}}\right] = \frac{1}{8}\frac{T_s}{RC} = \frac{e_s}{8}$$

$$\therefore \qquad e_d = \frac{e_s}{8}$$

This proves that $\quad e_d = \dfrac{e_s}{8} = \dfrac{e_t}{4} \quad$ or $\quad e_s = 2e_t = 8e_d$

If a capacitor *C is charged by a constant current I, then the voltage across C is It/C. Hence the rate of change of voltage with time is given by Sweep speed = I/C*

UNIJUNCTION TRANSISTOR

As the name implies a UJT has only one p-n junction, unlike a BJT which has two p-n junctions

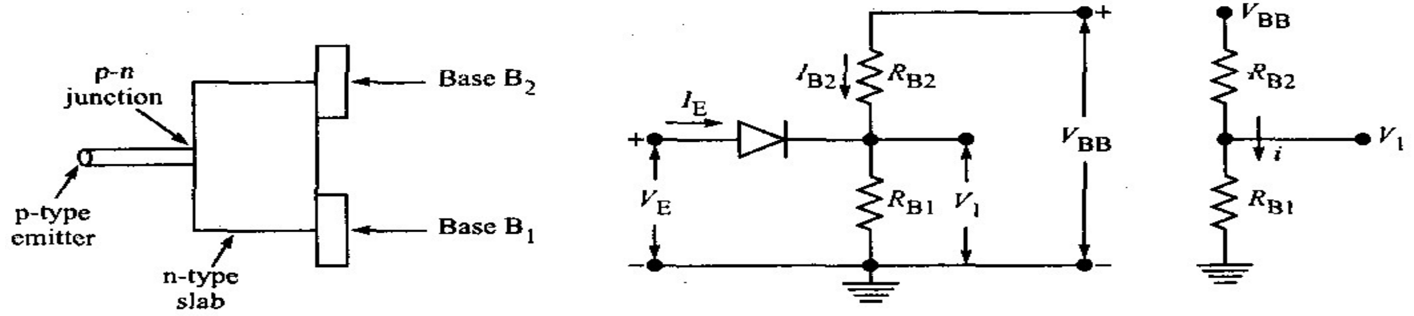
It has a p-type emitter alloyed to a lightly doped n-type material as shown in Figure 5.4(a).

There are two bases: base B1 and base B2, base B1 being closer to the emitter than base B2. The p-n junction is formed between the p-type emitter and n-type silicon slab.

Originally this device was named as double base diode but now it is commercially known as UJT.

The equivalent circuit of the UJT is shown in Figure 5.4(b).

R_{B1} is the resistance between base B1 and the emitter, and it is basically a variable resistance, its value being dependent upon the emitter current i_E. R_{B2} is the resistance between base B_2 and the emitter, and its value is fixed.



(a) (b) (c)

Figure 5.4 (a) Construction of UJT, (b) equivalent circuit of UJT, and (c) circuit when iE = 0.

If IE = 0, due to the applied voltage VBB, a current i results as shown in Figure 5.4(c).

$$i = \frac{V_{BB}}{R_{B1} + R_{B2}}$$

$$V_1 = iR_{B1} = \frac{V_{BB}}{R_{B1} + R_{B2}} R_{B1} = \frac{R_{B1}}{R_{B1} + R_{B2}} V_{BB}$$

The ratio $\frac{R_{B1}}{R_{B1} + R_{B2}}$ is termed the *intrinsic stand off ratio* and is denoted by η. Therefore,

$$\eta = \frac{R_{B1}}{R_{B1} + R_{B2}} \quad \text{when } i_E = 0.$$

$$V_1 = \eta V_{BB}$$

From the equivalent circuit, it is evident that the diode cannot conduct unless the emitter voltage $V_E = V_\gamma + V_1$ where V_γ is the cut-in voltage of the diode

This value of emitter voltage which makes the diode conduct is termed *peak voltage and is denoted by VP.*

$$V_P = V_\gamma + V_1$$

$$V_P = V_\gamma + \eta V_{BB} \quad \text{since } V_1 = \eta V_{BB}$$

It is obvious that if *VE < VP, the UJT is OFF and if VE > VP, the UJT is ON.*

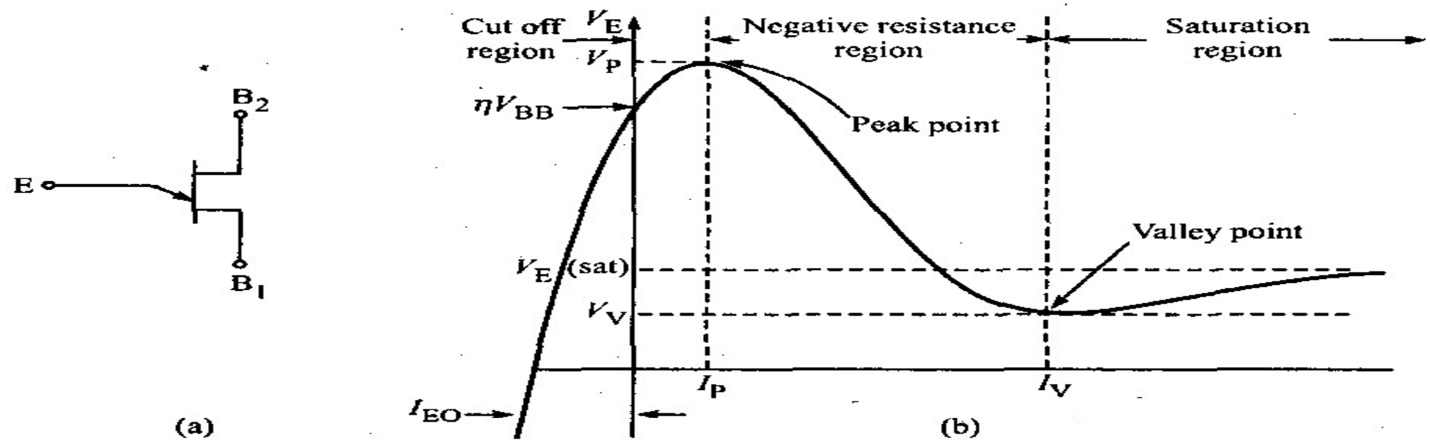


Figure 5.5 (a) Symbol and (b) input characteristics of UJT.

The symbol of UJT is shown in Figure 5.5(a). The input characteristics of UJT (plot of VE versus i_E) are shown in Figure 5.5(b).

The main application of UJT is in switching circuits wherein rapid discharge of capacitors is very essential. UJT sweep circuit is called a relaxation oscillator.

Applications of UJT

UJTs are most prominently used as relaxation oscillators. They are also used in Phase Control Circuits. In addition, UJTs are widely used to provide clock for digital circuits, timing control for various devices, controlled firing in thyristors, and sync pulsed for horizontal deflection circuits in CRO.

SWEEP CIRCUIT USING UJT

Many devices are available to serve as the switch.

Figure 5.6(a) shows the exponential sweep circuit in which the UJT serves the purpose of the switch.
In fact, any current-controlled negative-resistance device may be used to discharge the sweep capacitor.

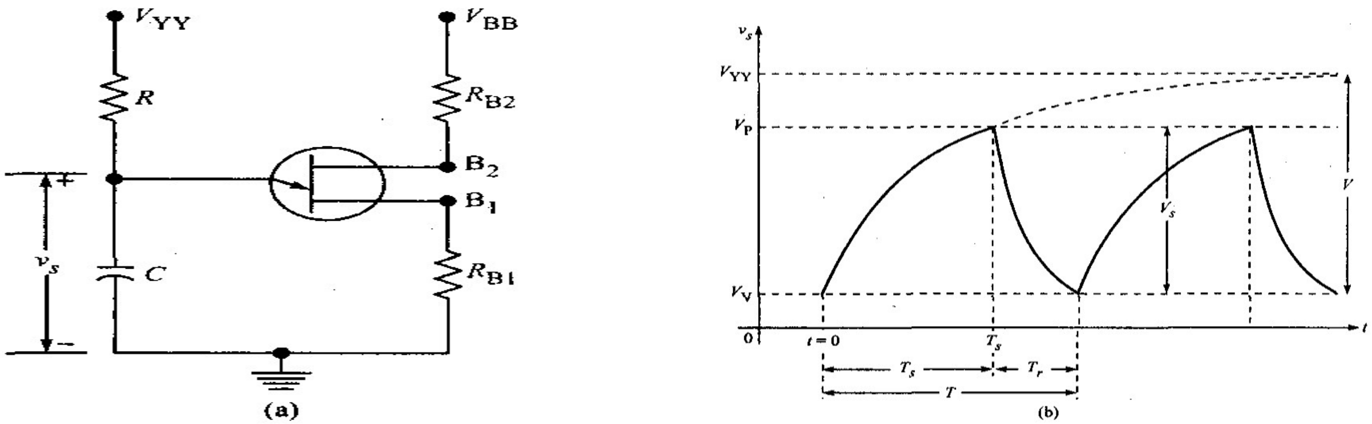


Figure 5.6 (a) UJT sweep circuit and (b) output waveform across the capacitor

The supply voltage Vyy and the charging resistor *R must be selected such that the load line intersects the input characteristic in the negative-resistance region.*

The UJT is OFF. The capacitor *C charges from V_{YY} through R. When it is charged to the peak value V_P, the UJT turns ON and the capacitor now discharges through the UJT.*

When the capacitor discharges to the valley voltage V_V ,tne UJT turns OFF, and again the capacitor starts charging and the cycle repeats.

The capacitor voltage appears as shown in Figure 5.6(b).

The expression for the sweep time *Ts can be obtained as follows.*

For $0 < t < T_s$, $v_s = V_{YY} - (V_{YY} - V_V)\, e^{-t/RC}$

At $t = T_s$, $v_o = v_s = V_P$

$\therefore$ $V_P = V_{YY} - (V_{YY} - V_V)e^{-T_s/RC}$

i.e. $(V_{YY} - V_V)e^{-T_s/RC} = V_{YY} - V_P$

or $$e^{T_s/RC} = \frac{V_{YY} - V_V}{V_{YY} - V_P}$$

$$\therefore \quad T_s = RC \ln \frac{V_{YY} - V_V}{V_{YY} - V_P}$$

Frequency of oscillation $f = \dfrac{1}{T}$

where $T = T_s + T_r$ is the period

$$\therefore \quad f = \frac{1}{T_s + T_r} = \frac{1}{T_s}, \text{ neglecting } T_r \text{ since } T_r << T_s.$$

$$\therefore \quad f = \frac{1}{RC \ln\left(\dfrac{V_{YY} - V_V}{V_{YY} - V_P}\right)} = \frac{1}{RC \ln\left(\dfrac{V_{YY}}{V_{YY} - V_P}\right)} \text{ since } V_V << V_{YY}$$

$$= \frac{1}{RC \ln\left(\dfrac{1}{1 - V_P / V_{YY}}\right)}$$

$$\text{Peak voltage } V_P = V_\gamma + \eta V_{BB}$$
$$= V_\gamma + \eta\, V_{YY}, \text{ if } V_{BB} \approx V_{YY}$$
$$= \eta\, V_{YY} \text{ neglecting } V_\gamma \text{ as } V_\gamma \text{ is} << V_{YY}$$

or $$\frac{V_P}{V_{YY}} = \eta, \text{ the intrinsic stand off ratio}$$

Substituting η for (V_P/V_{YY}), we get

$$f = \frac{1}{RC \ln\left(\dfrac{1}{1-\eta}\right)}$$

For good linearity, $V_s = V_P - V_V$ must be much smaller than $V = V_{YY} - V_V$. Since usually $V_P >> V_V$ and $V_{YY} >> V_V$, we require that $V_P << V_{YY}$. Also, $V_{YY} >> V_{BB}$.

When V_V is very small,

$$e_s = \frac{T_s}{RC}, \quad e_t = \frac{T_s}{2RC} \quad \text{and} \quad e_d = \frac{T_s}{8RC}$$

MILLER AND BOOTSTRAP TIME-BASE GENERATORS—BASIC PRINCIPLES

The linearity of the time-base waveforms may be improved by using circuits involving feedback.

Figure 5.10(a) shows the basic exponential sweep circuit in which S opens to form the sweep.

A linear sweep cannot be obtained from this circuit because as the capacitor charges, the charging current decreases and hence the rate at which the capacitor charges, i.e. the slope of the output waveform decreases.

A perfectly linear output can be obtained if the initial charging current I = *V/R is maintained constant. This can be done by introducing an auxiliary variable generator v whose generated voltage v is always equal to and opposite to the voltage across the capacitor as shown in Figure 5.10(b).*

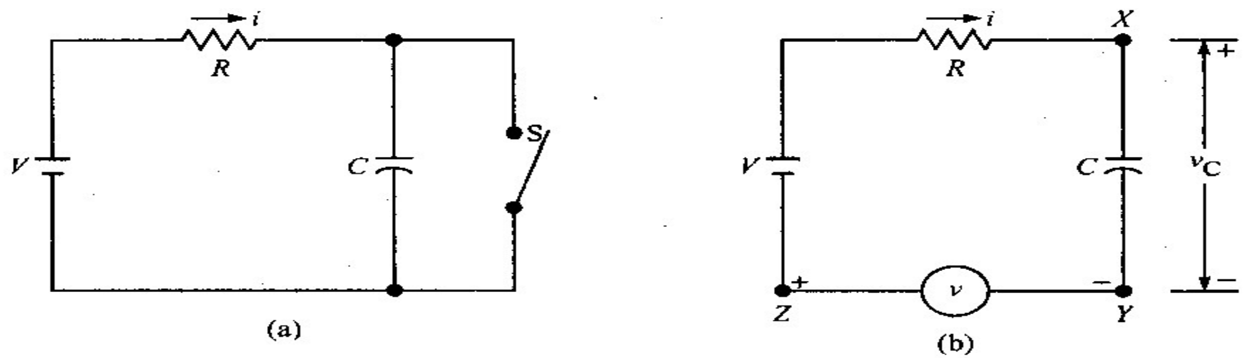


Figure 5.10 (a) The current decreases exponentially with time and (b) the current remains constant

The circuit of Figure 5.10(b) suppose the point Z is grounded as in Figure 5.11(a).
A linear sweep will appear between the point Y and ground and will increase in the negative direction.
Let us now replace the fictitious (imaginary) generator by an amplifier with output terminals YZ and input terminals XZ as shown in Figure 5.11(b).
Since we have assumed that the generated voltage is always equal and opposite to the voltage across the capacitor, the voltage between X and Z is equal to zero. Hence the point X acts as a virtual ground. Now for the amplifier, the input is zero volts and the output is a finite negative value.

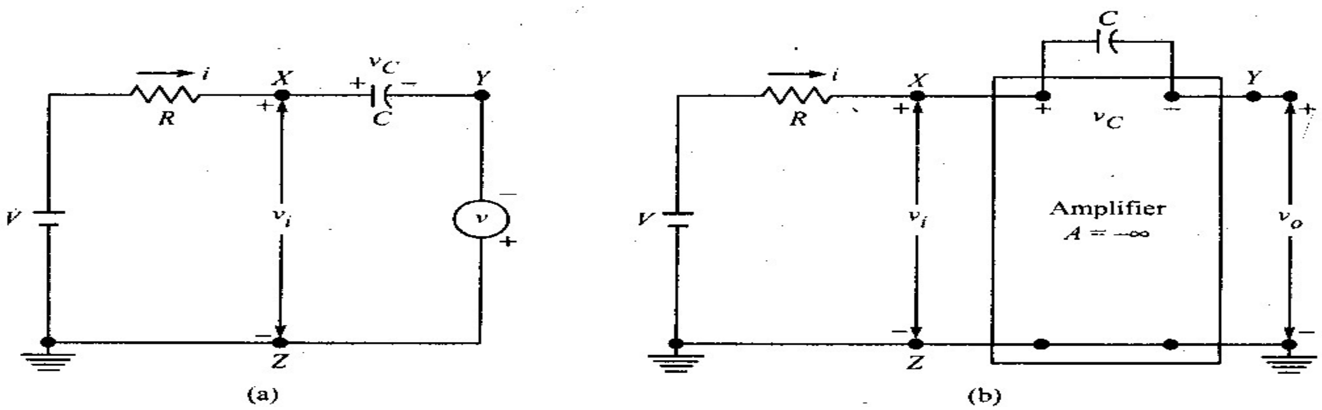
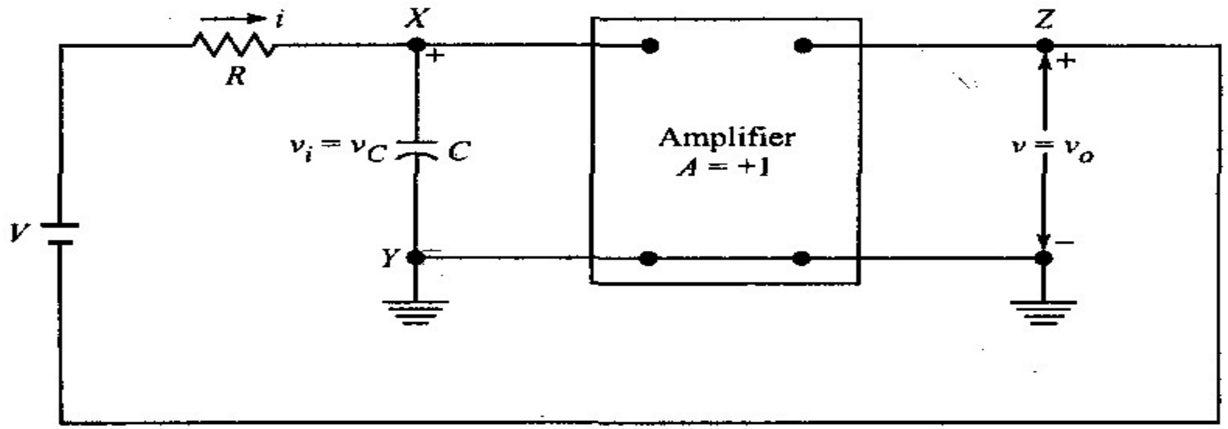


Figure 5.11 (a) Figure 5.10(b) with Z grounded and (b) Miller integrator circuit.

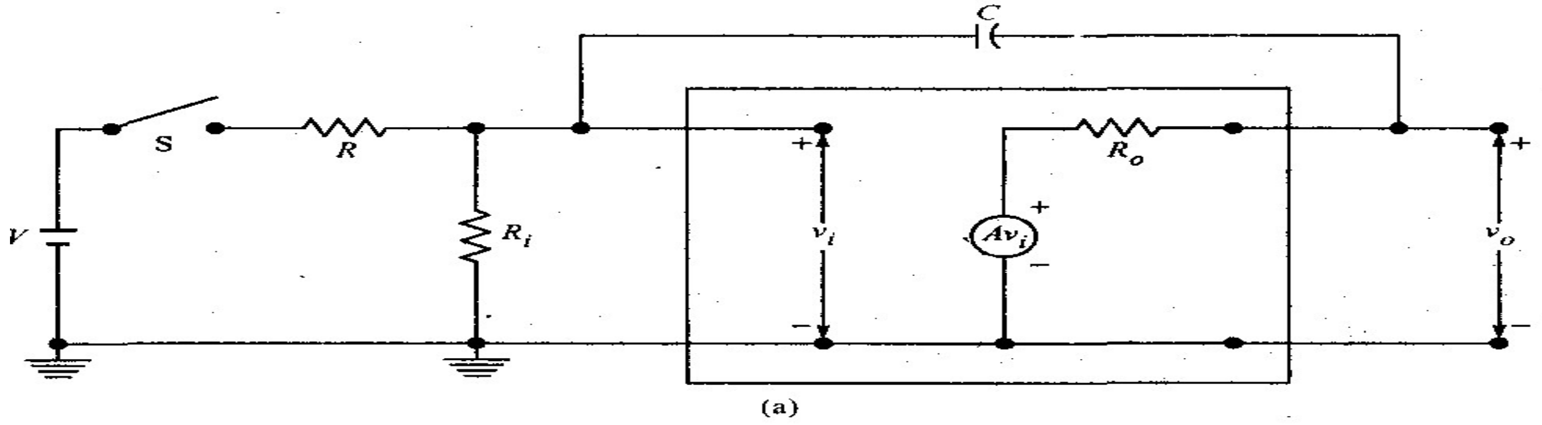
This can be achieved by using an operational integrator with a gain of infinity. This is normally referred to as the Miller integrator circuit or the Miller sweep.

Suppose that the point Y in Figure 5.10(b) is grounded and the output is taken at Z. A linear sweep will appear between Z and ground and will increase in the positive direction.

Let us now replace the fictitious generator by an amplifier with input terminals XY and output terminals ZY as shown in Figure 5.12. Since we have assumed that the generated voltage v at any instant is equal to the voltage across the capacitor vc, then *v0 must be equal to* v_i, *and the amplifier voltage gain must be equal to unity. The circuit of Figure 5.12 is referred to as the Bootstrap sweep circuit.*



The Miller sweep

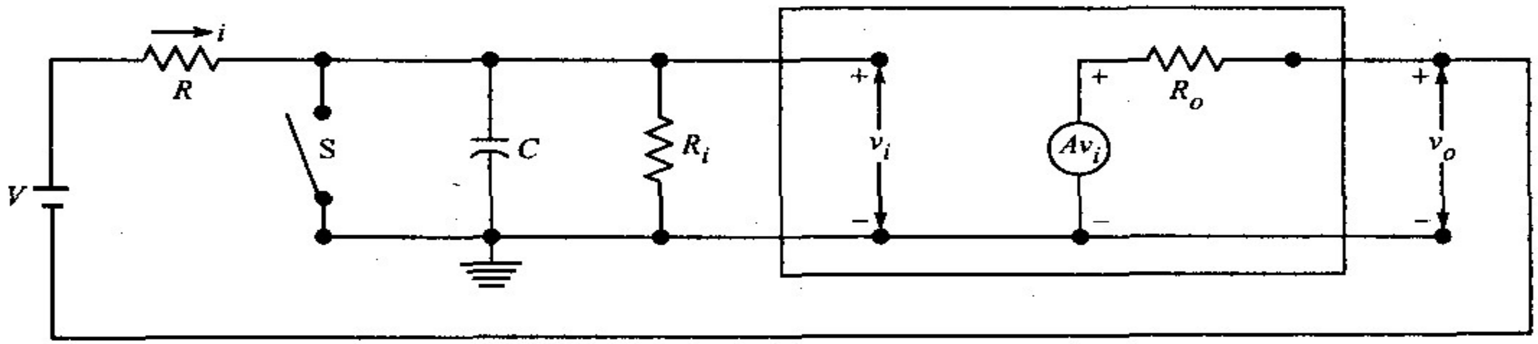


$$v_i(t = 0^+) = \Delta v_i = v_o(t = 0^+) = \Delta v_o = \frac{\left(\frac{R_o}{R'}\right) V'}{1 - A + \frac{R_o}{R'}}$$

$$v_i(t = 0^+) \approx \frac{R_o V'}{R' |A|}$$

if *R0 is taken into account, v0(t = 0+) is a small positive value and still it will be a negative-going sweep with the same terminal value. Thus the negative-going ramp is preceded by a small positive jump*

The bootstrap sweep



$$v_o(t = \infty) = AV_i - iR_o = AiR_i - iR_o = i(AR_i - R_o)$$

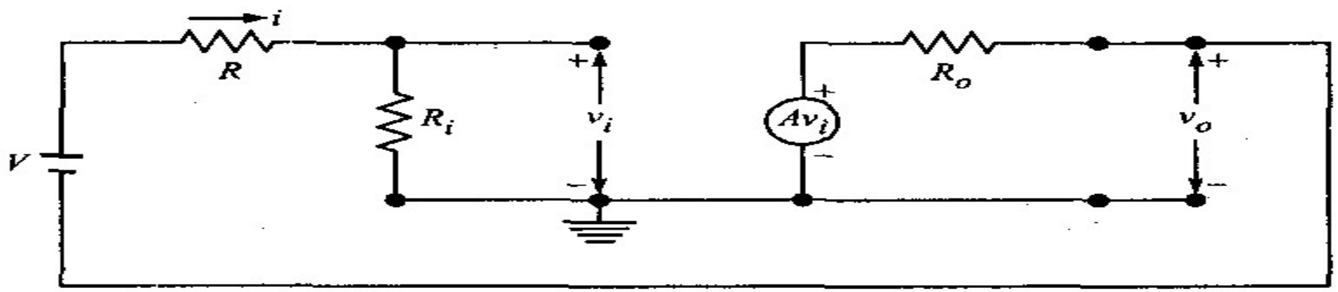


Figure 5.16 Equivalent circuit of Figure 5.14 at $t = \infty$.

$$V - iR - iR_i + AV_i - iR_o = 0$$

i.e.

$$i = \frac{V}{R + R_o + R_i(1 - A)}$$

$\therefore$

$$v_o(t = \infty) = \frac{V(AR_i - R_o)}{R + R_o + R_i(1 - A)}$$

THE TRANSISTOR MILLER TIME-BASE GENERATOR

The transistor Miller time base generator circuit is the popular **Miller integrator** circuit that produces a sweep waveform. This is mostly used in horizontal deflection circuits. Let us try to understand the construction and working of a Miller time base generator circuit.

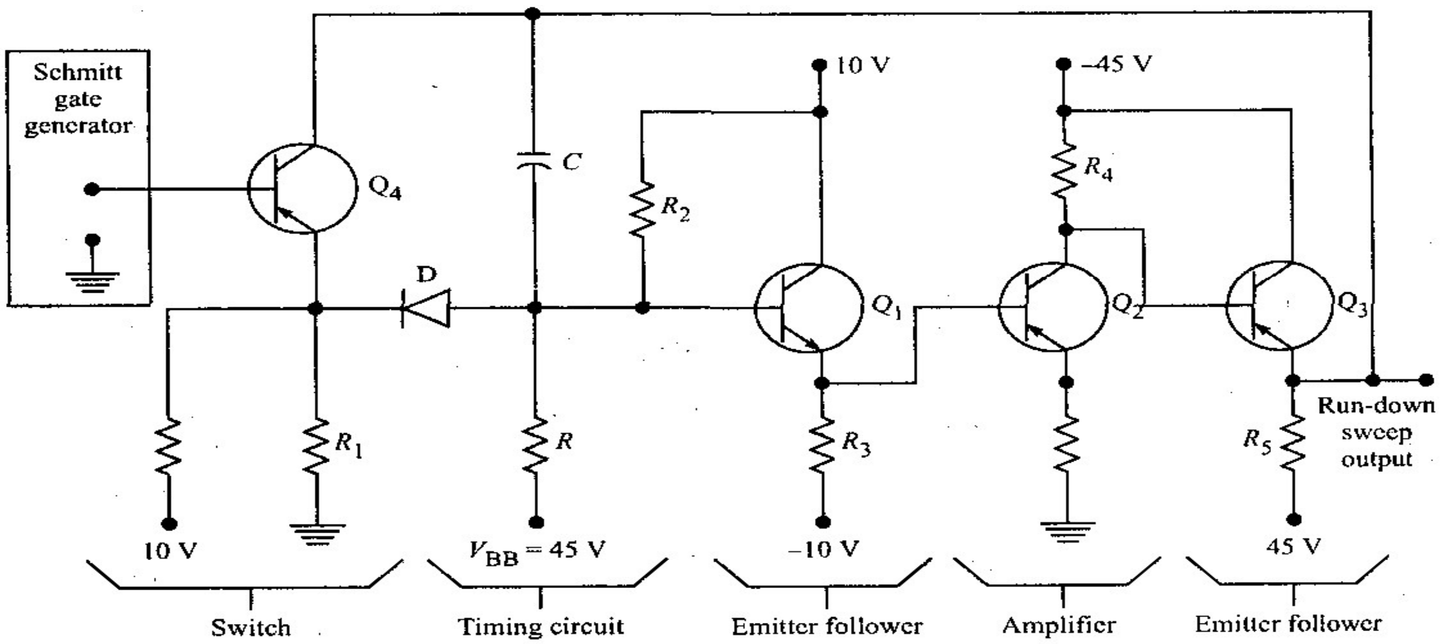


Figure 5.17 A transistorized Miller time-base generator.

$$e_s = \frac{V_s}{V}\left(1 - A + \frac{R}{R_i} + \frac{C}{C_1}\right)$$

The Miller time base generator circuit consists of a switch and a timing circuit in the initial stage, whose input is taken from the Schmitt gate generator circuit. The amplifier section is the following one which has three stages, first being an **emitter follower**, second an **amplifier** and the third one is also an **emitter follower**.

An emitter follower circuit usually acts as a **Buffer amplifier**. It has a **low output impedance** and a **high input impedance**. The low output impedance lets the circuit drive a heavy load. The high input impedance keeps the circuit from not loading its previous circuit. The last emitter follower section will not load the previous amplifier section. Because of this, the amplifier gain will be high.

The capacitor C placed between the base of Q_1 and the emitter of Q_3 is the timing capacitor. The values of R and C and the variation in the voltage level of V_{BB} changes the sweep speed. The figure below shows the circuit of a Miller time base generator.

When the output of Schmitt trigger generator is a negative pulse, the transistor Q_4 turns ON and the emitter current flows through R_1. The emitter is at negative potential and the same is applied at the cathode of the diode D, which makes it forward biased. As the capacitor C is bypassed here, it is not charged.

The application of a trigger pulse, makes the Schmitt gate output high, which in turn, turns the transistor Q_4 OFF. Now, a voltage of 10v is applied at the emitter of Q_4 that makes the current flow through R_1 which also makes the diode D reverse biased. As the transistor Q_4 is in cutoff, the capacitor C gets charged from V_{BB} through R and provides a rundown sweep output at the emitter of Q_3. The capacitor C discharges through D and transistor Q_4 at the end of the sweep.

Considering the effect of capacitance C_1, the slope speed or sweep speed error is given by

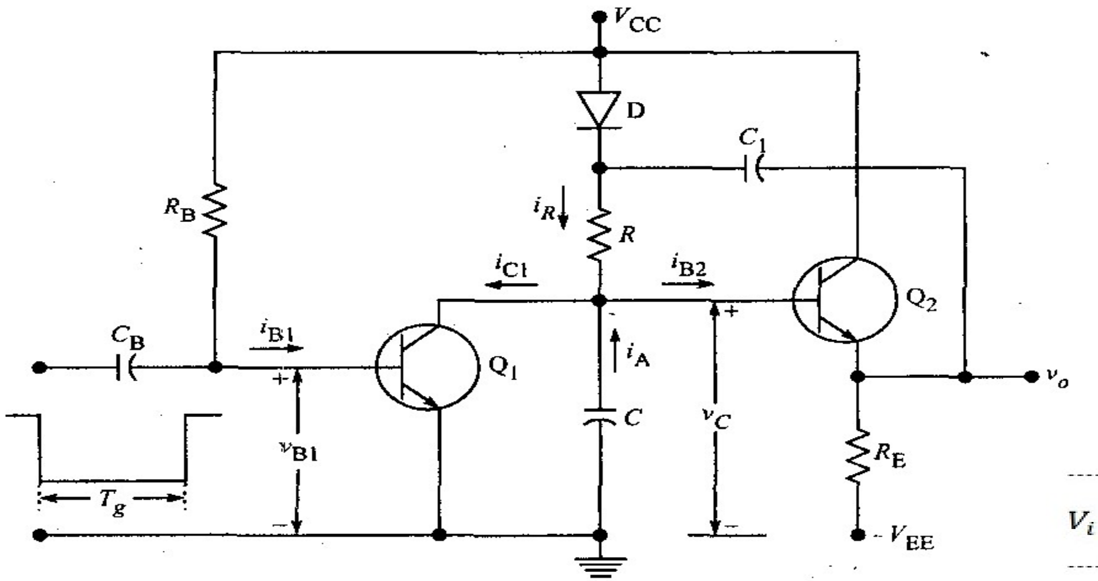
$$e_s = \frac{V_s}{V}\left(1 - A + \frac{R}{R_i} + \frac{C}{C_1}\right)$$

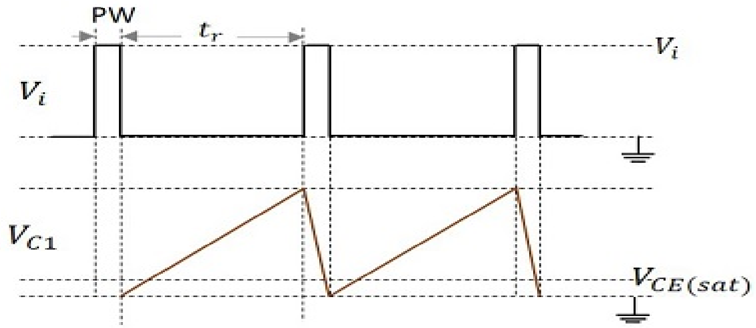
Applications

Miller sweep circuits are the most commonly used integrator circuit in many devices. It is a widely used saw tooth generator.

THE TRANSISTOR BOOTSTRAP TIME-BASE GENERATOR

A bootstrap sweep generator is a time base generator circuit whose output is fed back to the input through the feedback. This will increase or decrease the input impedance of the circuit. This process of **bootstrapping** is used to achieve constant charging current.





Before the application of gating waveform at t = 0, as the transistor gets enough base drive from V_{CC} through R_B, Q_1 is ON and Q_2 is OFF. The capacitor C_2 charges to V_{CC} through the diode D.

Then a negative trigger pulse from the gating waveform of a Monostable Multivibrator is applied at the base of Q_1 which turns Q_1 OFF. The capacitor C_2 now discharges and the capacitor C_1 charges through the resistor R. As the capacitor C_2 has large value of capacitance, its voltage levels (charge and discharge) vary at a slower rate. Hence it discharges slowly and maintains a nearly constant value during the ramp generation at the output of Q_2.

During the ramp time, the diode D is reverse biased. The capacitor C_2 provides a small current I_{C1} for the capacitor C_1 to charge. As the capacitance value is high, though it provides current, it doesn't make much difference in its charge. When Q_1 gets ON at the end of ramp time, C_1 discharges rapidly to its initial value. This voltage appears across V_O. Consequently, the diode D gets forward biased again and the capacitor C_2 gets a pulse of current to recover its small charge lost during the charging of C_1. Now, the circuit is ready to produce another ramp output.

The capacitor **C2** which helps in providing some feedback current to the capacitor C1 acts as a **boot strapping capacitor** that provides constant current.

Advantage

The main advantage of this boot strap ramp generator is that the output voltage ramp is very linear and the ramp amplitude reaches the supply voltage level.

Current Time base Generator

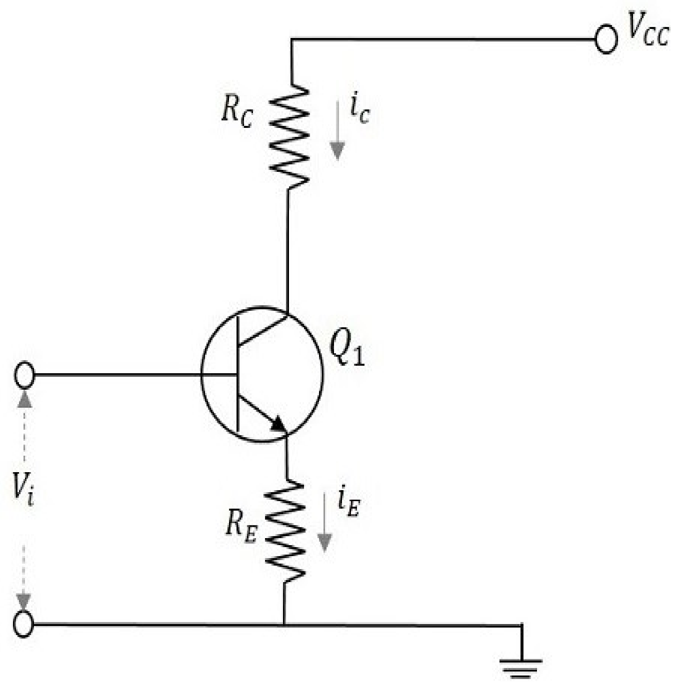
A time base generator that provides an output current waveform that varies linearly with time is called as a Current Time base Generator.

Let us try to understand the basic current time base generator.

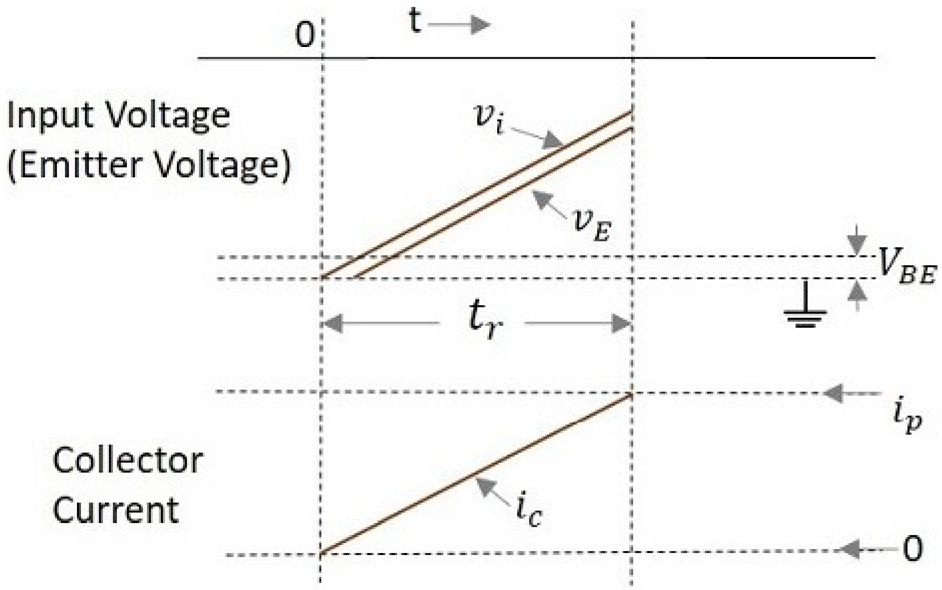
A basic simple RC time base generator or a Ramp generator or a sweep circuit consists of a common-base configuration transistor and two resistors, having one in emitter and another in collector. The V_{CC} is given to the collector of the transistor. The circuit diagram of a basic ramp current generator is as shown here under.

A transistor connected in common-base configuration has its collector current vary linearly with its emitter current. When the emitter current is held constant, the collector current also will be near constant value, except for very smaller values of collector base voltages.

As the input voltage V_i is applied at the base of the transistor, it appears at the emitter which produces the emitter current i_E and this increases linearly as V_i increase from zero to its peak value. The collector current increases as the emitter current increases, because i_C is closely equal to i_E.



The input and output waveforms are as shown below.



Sampling Gates

Sampling Gates are also called as Transmission gates, linear gates and selection circuits, in which the output is exact reproduction of the input during a selected time interval and zero otherwise

The time interval for transmission is selected by an externally impressed signal called gating signal

These are two types

Unidirectional

Bidirectional

Sampling gates are different from the logic gates. In logic gates can be any number of Inputs and outputs and output is not exactly reproduction of the input.

Output of sampling gate is exactly reproduction of input (whatever the shape of input square, sine, pulse etc) during selected period.

Principle of operation of a linear gate

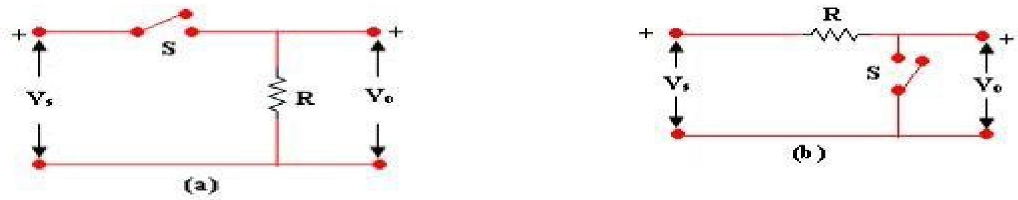
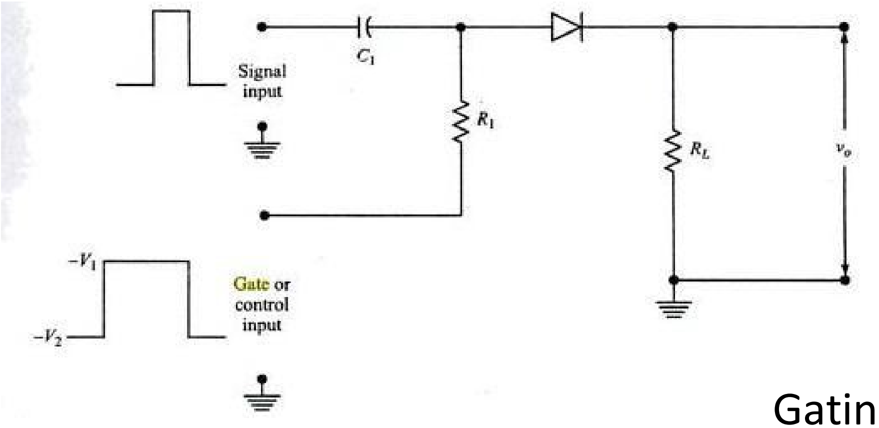


Fig. Linear gates

In (a) the switch closes for transmitting the signal whereas in (b) the switch is open for transmission to take place.

Unidirectional Gate

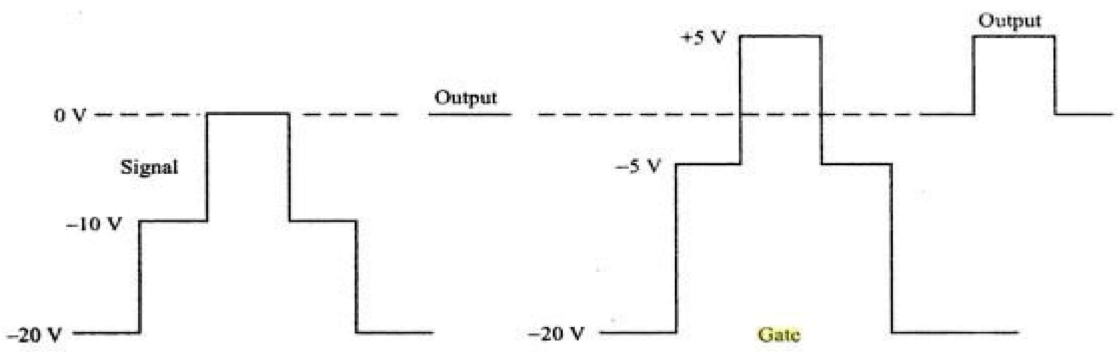


unidirectional sampling gates are those which transmit signals of only one polarity(i.e,. either positive or negative)

Gating signal determines transmission period

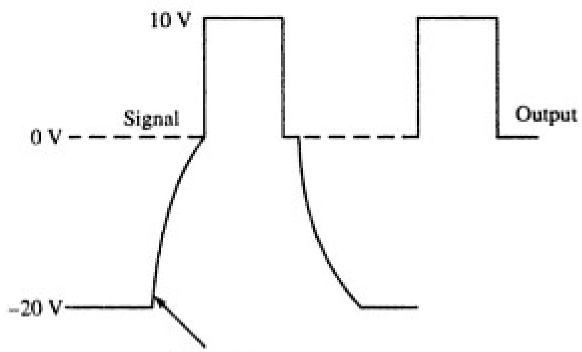
The gating signal is also known as control pulse, selector pulse or an enabling pulse. It is a negative signal, the magnitude of which changes abruptly between –V2 and–V1.

Outputwaveform



pedestal

When the control signal is shifted to positive value ,so it will be superimposed on input and control signals .so the pedestal occurs



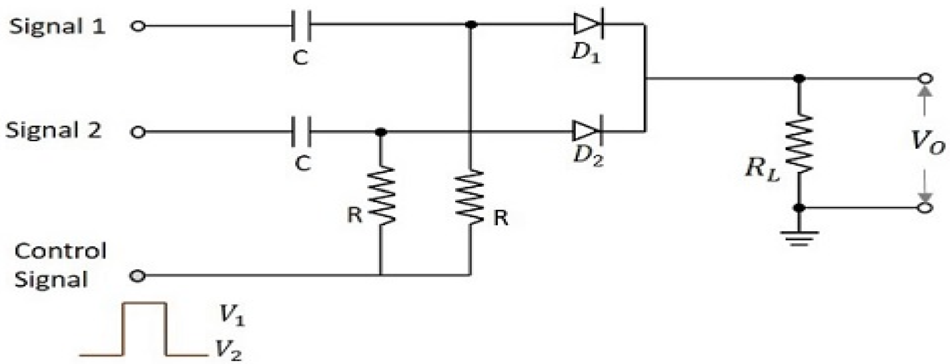
The advantages of Unidirectional sampling gate

It is simple
There is very little time delay through gate
The gate draws no current in its quiescent conditions
The gate can be easily extended

Disadvantages

There's interaction between control and input signals (V_C and V_S)
As the number of inputs increase, the loading on control input increases.
Output is sensitive to control input voltage V_1 (upper level of V_C)
Only one input should be applied at one instant of time.
Because of slow rise time of the control signal, the output may get distorted, if the input signal is applied before reaching the steady state.

Unidirectional sampling gate more than one input



When the control input is given,
At $V_C = V_1$ which is during the transmission period, both the diodes D_1 and D_2 are forward biased.
Now, the output will be the sum of all the three inputs.

$$VO=VS1+VS2+VC$$

For V_1 = 0v which is the ideal value,

$$VO=VS1+VS2$$

Here we have a major limitation that at any instant of time, during the transmission period, only one input should be applied. This is a disadvantage of this circuit.
During the non-transmission period,

$$VC=V2$$

Both the diodes will be in reverse bias which means open circuited.
This makes the output $VO=0V$
The main disadvantage of this circuit is that the **loading on the control signal** increases as the number of inputs increase. This limitation can be avoided by another circuit in which the control input is given after the input signal diodes.

Unidirectional diode sampling gate with multiple gating signals

A unidirectional diode coincidence gate is shown below.

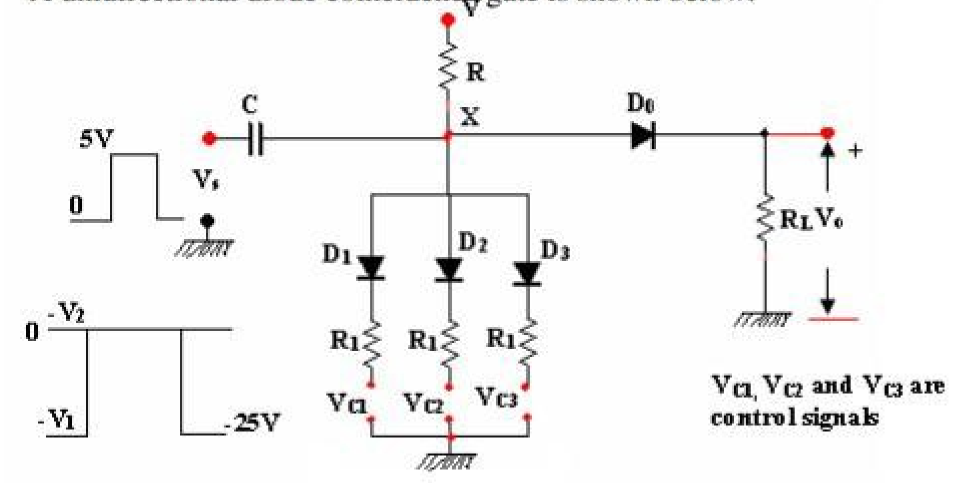
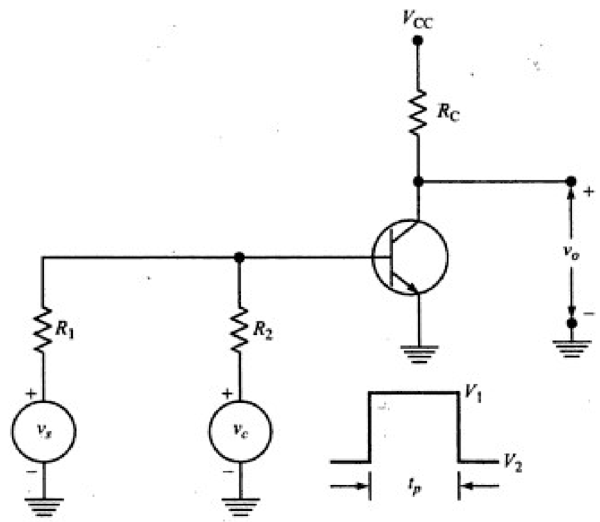
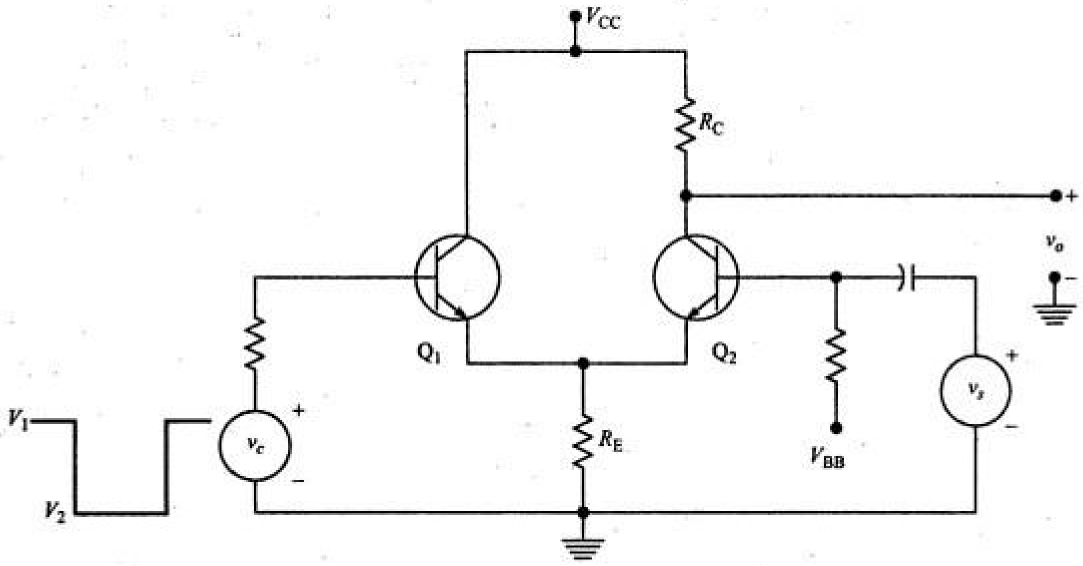


Fig. A unidirectional diode AND gate

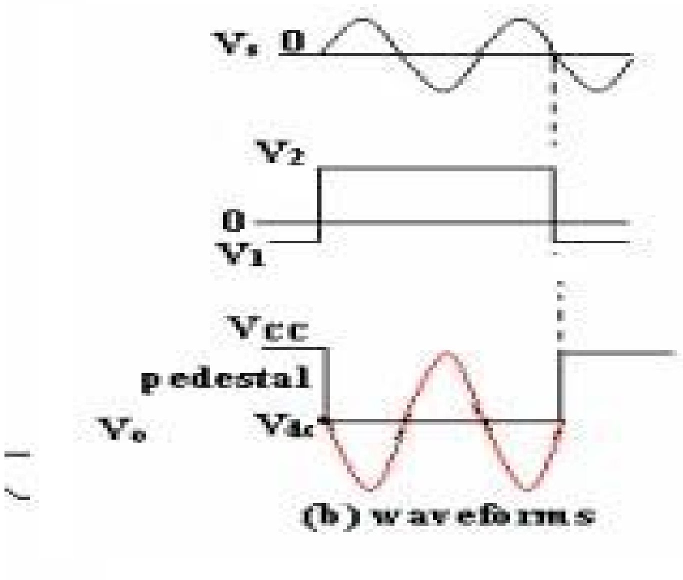
Bidirectional Sampling gate:



Emitter coupled Bidirectional Sampling gate using Transistor



In bidirectional sampling gates there exit Pedastal

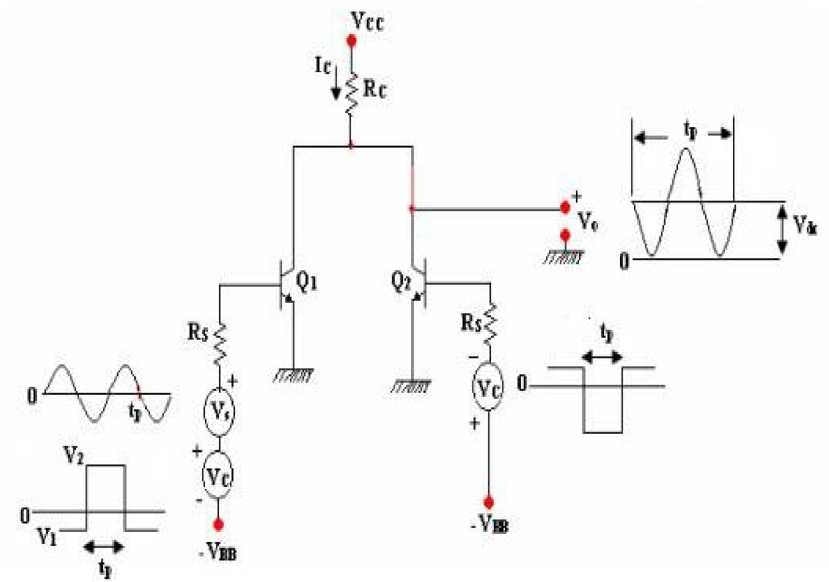


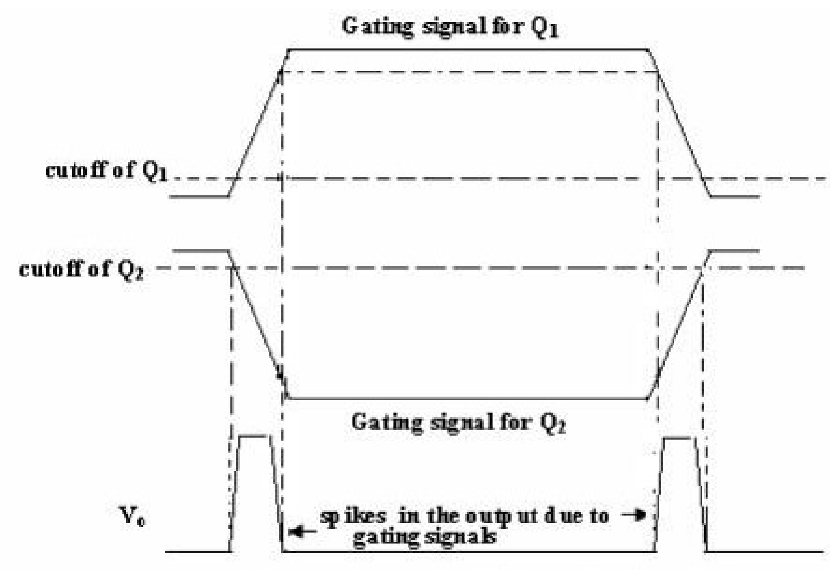
Circuit that minimizes the pedestal

The difference in the output signals during transmission period and non-transmission period though the input signals is not applied, is called as **Pedestal**. It can be a positive or a negative pedestal.

Hence it is the output observed because of the gating voltage though the input signal is absent. This is unwanted and has to be reduced. The circuit below is designed for the reduction of pedestal in a gate circuit.

A circuit arrangement that reduces this pedestal is shown in fig.





(a) when the rise time of the gating signal is large

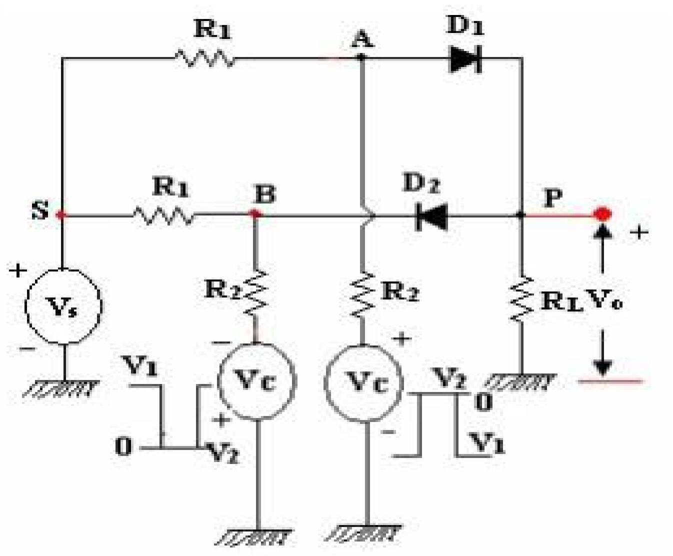
When the control signal is applied, during the transmission period i.e. at V_1, Q_1 turns ON and Q_2 turns OFF and the V_{CC} is applied through R_C to Q_1.

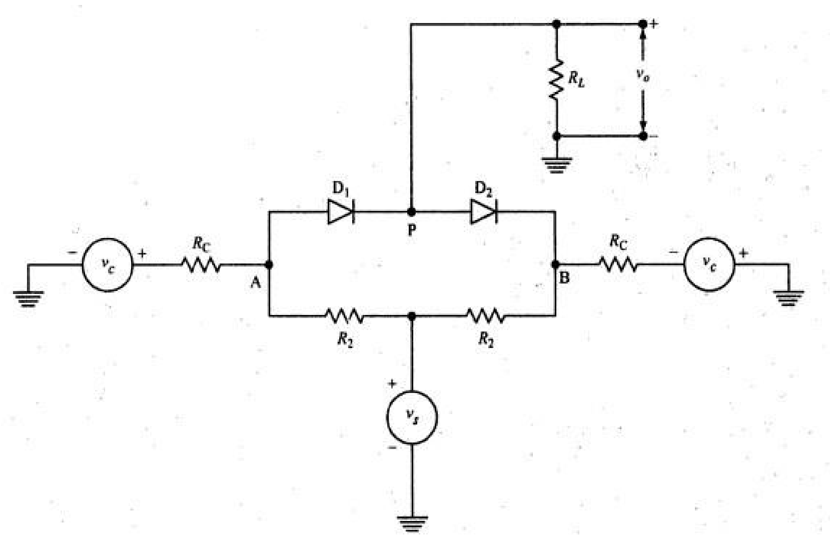
Whereas during the non transmission period i.e. at V_2, Q_2 turns ON and Q_1 turns OFF and the V_{CC} is applied through R_C to Q_2. The base voltages $-V_{BB1}$ and $-V_{BB2}$ and the amplitude of gate signals are adjusted so that two transistor currents are identical and as a result the quiescent output voltage level will remain constant.

If the gate pulse voltage is large compared with the V_{BE} of the transistors, then each transistor is biased far below cut off, when it is not conducting. So, when the gate voltage appears, Q_2 will be driven into cut off before Q_1 starts to conduct, whereas at the end of the gate, Q_1 will be driven to cut off before Q_2 starts to conduct.

Hence the gate signals appear as in the above figure. The gated signal voltage will appear superimposed on this waveform. These spikes will be of negligible value if the gate waveform rise time is small compared with the gate duration.

Two Diode Sampling gate



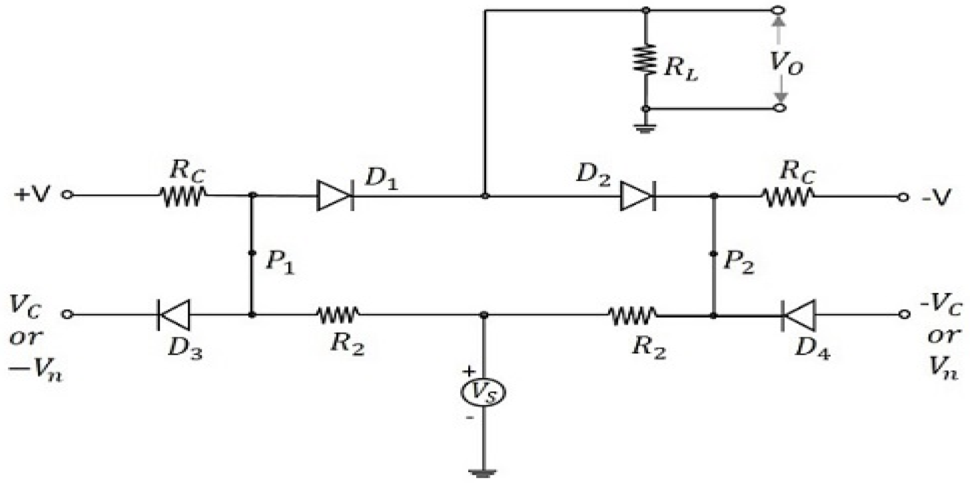


Four Diode Bidirectional Sampling Gate

Bidirectional sampling gate circuit is made using diodes also. A two diode bidirectional sampling gate is the basic one in this model. But it has few disadvantages such as

- It has low gain
- It is sensitive to the imbalances of control voltage
- $V_{n\ (min)}$ may be excessive
- Diode capacitance leakage is present.

A four diode bidirectional sampling gate was developed, improving these features. A two bidirectional sampling gate circuit was improved adding two more diodes and two balanced voltages +v or –v to make the circuit of a four diode bidirectional sampling gate as shown in the figure



The gain A of the circuit is given by

$$A = RC/(RC+R2) \times RL/\ RL+(Rs/2)$$

Applications of Sampling Gates

- •Sampling scopes
- •Multiplexers
- •Sample and hold circuits
- •Digital to Analog Converters
- •Chopped Stabilizer Amplifiers

Among the applications of sampling gate circuits, the Sampling scope circuit is prevalent.